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ABSTRACT

This report contains papers given in the third workshop on the Current State of Research on Mathematical Beliefs. No plenary talks were given. The presentations were categorized into the subjects of pupil beliefs and teacher beliefs. The concept of belief in this workshop also refers to conceptions, views, and attitudes. Pupils' beliefs and their connections to mathematics learning were mainly addressed within the framework of comprehensive school where the central theme seemed to be the development of pupils' beliefs in school. The international comparison was also a topic of research. In the case of teachers' beliefs, the central field of interest was the development of student teachers' beliefs. Papers include: (1) "Investigation of Motivation in Hungary" (Andras Ambrus); (2) "Mathematics versus Computer Science: Teachers' Views on Teacher Roles and the Relations of Both Subjects" (Peter Berger); (3) "A Theoretical Framework for Teachers' Conceptions" (Fulvia Furinghetti); (4) "Beliefs of Pupils and First Year Students about Mathematics Education" (Gunter Graumann); (5) "Feminine Structures in Mathematical Beliefs and Performances" (Markku Hannula and Marja-Liisa Malmivuori); (6) "Research Project: Development of Pupils Mathematical Beliefs" (Markku Hannula, Marja-Liisa Malmivuori, and Erkki Pehkonen); (7) "Mathematical Beliefs of Eighth-Graders: What is Mathematics?" (Kirsti Hoskonen); (8) "Prospective Teachers' Math Views and Educational Memories" (Sinikka Lindgren); (9) "Causes of Being Bad in Mathematics as Seen by Pupils of the German Gymnasium" (Christoph Oster); (10) "Some Findings in the International Comparison of Pupils' Mathematical Views" (Erkki Pehkonen); (11) "Some Observations Concerning Pupils' Views on Mathematics Teaching in Finland and Tatarstan (Russia)" (Erkki Pehkonen and Ildar S. Safuanov); (12) "Changing Pre-Service Teachers Attitudes towards Mathematics" (George Philippou and Constantinos Christou); and (13) "Change in Mathematical Views of First Year University Students" (Gunter Torner and Iris Kalesse). (Author/ASK/NB)

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Research Report 170

Erkki Pehkonen (ed.)

Current State of Research on Mathematical Beliefs III

Proceedings of the MAVI-3 Workshop August 23 – 26, 1996

Department of Teacher Education
University of Helsinki

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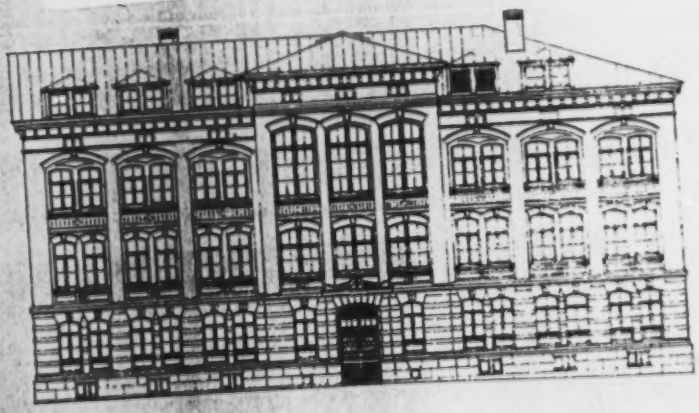

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Erkki Pehkonen (ed.)

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Erkki Pehkonen (ed.)

Current State of Research on Mathematical Beliefs III
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Abstract

The third workshop on Current State of Research on Mathematical Beliefs took place in the Department of Teacher Education at the University of Helsinki from the 23rd of August to the 26th of August 1996. This report contains most of the papers given in the workshop. The conference language was English.

There was no plenary talks, but every presentation had a time slot of 30 min with a follow-up discussion of another 30 min. The presentations were categorized as follows: During the first two days, pupils' beliefs were mainly dealt with, whereas the topic of the other two days was teachers' beliefs. Here the concept "belief" contains also conceptions, views and attitudes.

Pupils' beliefs and their connections to mathematics learning were mainly dealt with within the framework of comprehensive school (Andras Ambrus, Markku Hannula, Kirsti Hoskonen, Marja-Liisa Malmivuori, Christoph Oster, Erkki Pehkonen, Ildar Safuanov). Here the central theme seemed to be the development of pupils' beliefs in school, but the international comparison was also a topic of research. In the case of teachers' beliefs, the central field of interest was the development and developing of student teachers' beliefs (Peter Berger, Fulvia Furinghetti, Günter Graumann, Sinikka Lindgren, George Philippou, Günter Törner).

Keywords: mathematical beliefs, conceptions, views, attitudes

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Tutkimuksia n:o 170, 1996

Erkki Pehkonen (toim.)

Matemaattisten uskomusten tutkimuksen nykytila III
Raportti MAVI-3 kokouksesta 23 – 26.8.1996. 97 s.

Tiivistelmä

Kolmas matemaattisten uskomusten tutkimuksen tilaa käsittelevä kokous pidettiin Helsingin yliopiston opettajankoulutuslaitoksessa elokuun 23. päivästä elokuun 26. päivään 1996. Tämä raportti sisältää useimmat kokouksen esityksistä. Kokouskieli oli englanti.

Kokouksessa ei ollut erikseen pääesitelmiä, vaan kaikille esityksille oli varattu aikaa 30 min, jota seurasi 30 min keskustelujakso. Esitykset oli jaksoteltu siten, että kahtena ensimmäisenä päivänä käsiteltiin pääasiassa oppilaiden uskomuksia, sen sijaan kahden jälkimmäisen päivän aiheena olivat opettajien uskomukset. Tässä käsite "uskomus" sisältää myös käsitykset, näkemykset ja asenteet.

Oppilaiden uskomuksia ja niiden yhteyksiä matematiikan oppimiseen käsiteltiin yleensä oppivelvollisuuskoulun puitteissa (Andras Ambrus, Markku Hannula, Kirsti Hoskonen, Marja-Liisa Malmivuori, Christoph Oster, Erkki Pehkonen, Ildar Safuanov). Keskeinen teema näytti olevan oppilaiden uskomusten kehittyminen kouluaikana, mutta myös kansainvälinen vertailu oli tutkimuksen kohteena. Opettajien uskomusten kohdalla muodostivat opettajiksi opiskelevien uskomusten kehittyminen ja kehittäminen keskeisen osa-alueen (Peter Berger, Fulvia Furinghetti, Günter Graumann, Sinikka Lindgren, George Philippou, Günter Törner).

Avainsanat: matemaattiset uskomukset, käsitykset, näkemykset, asenteet

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Preface

The third Finnish-German workshop on Current State of Research on Mathematical Beliefs, the so-called MAVI-3 workshop took place in the Department of Teacher Education at the University of Helsinki from Friday the 23rd of August to Monday the 26th of August 1996. There were 21 participants of whom almost everybody had a presentation. This volume contains the abstracts of most of the talks given at the workshop.

In this report, every author is responsible for his / her own text. These are neither proof-read by the editor, nor their language is checked. Addresses of the contributors can be found in the appendix.

The Finnish-German research group MAVI (MAthematical VIews) is initiative of my colleague Günter Törner and myself, and its aim is to study and examine those mathematical-didactical questions which arise through research on mathematical beliefs. The two earlier workshops (MAVI-1 and MAVI-2) were organized at the University of Duisburg on October 1995 and March 1996 resp. Their proceedings are published in the Pre-Print -series of the mathematical institution at the University of Duisburg (nr. 310 and 340). The next workshop is planned to take place at the University of Duisburg on March 1997.

In this place, I want to thank our graduate students for the help I got when organizing the workshop: Markku Hannula kept records before the meeting on the participants, put together the preliminary program as well as organized the excursion on Sunday morning. Marja-Liisa Malmivuori and Riitta Soro took care of the coffee breaks. Furthermore, I like to express my gratitude to the Finnish Academy and the German DAAD for the financial support of our Finnish-German cooperation. My thanks are also due to our Head of Department, prof. Irina Koskinen for allowing me to publish these proceedings in the Research Report -series of the Department of Teacher Education.

Helsinki, October 1996

Erkki Pehkonen

Investigation of motivation in Hungary

Andras Ambrus
University of Budapest

Introduction

The investigation of the attitude, beliefs and motivation in mathematics education has no tradition in Hungary. In years 1990-91 Erkki Pehkonen and Klara Tompa made a comparative study about the mathematical views held by Finnish and Hungarian seventh-graders (Pehkonen & Tompa 1994). Two investigations were made by two mathematics teacher-students at the Eötvös Lorand University in 1996. E. Vörösvari and T. Tesenyi performed their investigations in the frame of their diplomwork. My presentation is based on the diplomwork of T. Tesenyi (1996).

Research objectives

The aim of investigation made by T. Tesenyi was to going nearer to answer the question "Why so many students in secondary school do not like the mathematics? What is their opinion about mathematics, about mathematics teaching and learning?" She have chosen conscious a 10th Grade secondary school class specializing in literature. She asked a 10th grade class in Apaczai Csere Janos Gymnasium Budapest to fill a questionnaire. The questionnaire is an extended, modified version of the questionnaire from H.J. Claus (1989). The questions together with the students' answers, we will present below. The questionnaire was anonymous, 16 students filled it. The students had the possibility to answer the question with yes or no. Some students at any questions could not choose from these alternatives, because they had an opinion between them. Another problem was the translation of the questionnaire from German to Hungarian. T. Tesenyi tried to give back the essence, the meaning of the questions instead using a direct translation. The same problem I have had at the translation from Hungarian to English. The statistics used was very simple, the number of the yes resp. no answers.

The questions and the student responses

1. Mathematics is between my 3 favorite subjects.	Yes: 6	No: 10
2. Mathematics is between my 3 last liked subjects.	Yes: 1	No: 15
3.If a would have possibility to choose the teaching subjects, I would choose mathematics.	Yes: 7	No: 7
4. If I would choose the teacher job, with pleasure would I teach mathematics	Yes: 2	No: 14

5. I think after the school I will have nothing to do with mathematics.	Yes: 7	No: 7
6. If I would have much time I would deal with mathematics as amusement.	Yes: 5	No: 11
7. I learn mathematics because		
a. the acquired knowledge is useful in the life.	Yes: 12	No: 4
b. I have success experiences at the solution of mathematical tasks.	Yes: 9	No: 7
c. I think it is interesting.	Yes: 11	No: 5
d. I want to reach that my teacher will be satisfied with me.	Yes: 6	No: 10
e. I want to get a good mark.	Yes: 14	No: 2
f. I would like to go to university after my secondary school studies and I need much point for it	Yes: 13	No: 3
g. I like mathematics	Yes: 12	No: 3
h. I am afraid from bad mark	Yes: 10	No: 6
9. I like in mathematics		
a. the computational tasks	Yes: 11	No: 5
b. the interesting mathematical problems.	Yes: 8	No: 8
c. everyday-life problems	Yes: 7	No: 9
10. I like to solve		
a. equations	Yes: 15	No: 1
b. word problems	Yes: 6	No: 10
c. inequalities	Yes: 4	No: 12
d. geometrical tasks	Yes: 4	No: 10
e. more difficult problems	Yes: 3	No: 13
11. I solve more likely 5 equations than one difficult problem.	Yes: 10	No: 5
12. On mathematics lessons I like		
a. to solve problems individually	Yes: 10	No: 6
b. to act at the chalkboard	Yes: 2	No: 14
c. to answer questions posed verbally	Yes: 4	No: 12
d. to work in group	Yes: 14	No: 2
e. to seek answers for questions need to think after	Yes: 8	No: 6
13. I am often afraid on mathematics lessons that I say something wrong.	Yes: 6	No: 10
14. At the written tests I always fear I make computational errors.	Yes: 11	No: 4
15. I fear very much from mathematics written tests.	Yes: 4	No: 12
16. I do not fear from mathematics written tests.	Yes: 10	No: 5
17. If I must go to the chalkboard on a mathematics lesson I am very nervous	Yes: 5	No: 10
18. Mathematics is easy for me, I do not have difficulties.	Yes: 3	No: 12
19.Mathematics is so difficult for me that I do not understand a lot of things.	Yes: 2	No: 13
20. I solve one difficult sequence of problems more likely alone.	Yes: 5	No: 11
21. I understand the material sometimes only with help of my parents resp. my classmates.	Yes: 11	No: 5
22. I understand the material often only then when my parents resp. classmates explain it for me.	Yes: 1	No: 15
23. Sometimes I can not solve the mathematical tasks without help.	Yes: 16	No: 0
24. Often can I not solve the mathematical test without help.	Yes: 0	No: 16
25. I learn only what interests me.	Yes: 6	No: 10

26. I learn the boring parts (things) too.	Yes: 10	No: 4
27. I am looking sometimes after the interesting but not compulsory things too, because I like to find answers for my problems	Yes:7	No: 9
28. My parents appreciate if I learn well (my achievement is good	Yes: 13	No: 2
29. I get reward at home for my good learning (achievement)	Yes: 3	No: 13

Reflection, interpretation of the results

One class with 16 students choosen from a good secondary school in Budapest does not allow to deduce generalizable consequences. Main aim of this work was to introduce T. Tesenyi into the pedagogical research work. The questionnaire was only small part of her diplomwork.

Some interesting characteristics of students' responses.

- *To give preference to mechanical, algorithmical procedures: solving of computational tasks resp. equations.*
- *Disliking the word problems, inequalities, geometrical tasks, in general more difficult problems.*
- *Prefering the individual and first of all the group work.*
- *The neglecting of the answering verbally posed questions and the acting at the chalkboard may influence in a negative manner the efficiency of the class discussions and students' explorations made at the chalkboard opposite to the whole class .*
- *Fear from the written tests, fear from the mistakes, computational errors.* (Unfortunately a lot of Hungarian secondary mathematics teachers share the opinion, there is no place for the errors in mathematics teaching)
- *The external motivation factors are dominant: good mark, points for the university study, appreciation of the teacher resp. parents, usefulness in the life.*
- *In a literature class we must consider as a very positive result, that most of the students learn mathematics because it is interesting, and most of them like mathematics. (internal motivation) . .*
- *All of the questioned students sometimes need help to solve mathematical problems, and most of the students hold mathematics as difficult.* (Do we need to teach so much and so abstract mathematics for all, as we make it in Hungary?)

Final remarks

The MAVI- 3 Meeting made quite clear for me, that we in Hungary need to change our opinion about some basic questions of mathematics teaching and learning. We need contact international projects not only in pure content and assessment questions of mathematics teaching but pedagogical, psychological projects, too.

There are some barriers which hinder to achieve this aim:

1. The very low acceptance of Pedagogy resp. Psychology between mathematicians and mathematics teachers in Hungary. For a long time - from 1945 to 1990 - these disciplines were overideologized by marxists-

leninists doctrines. It was a general opinion: mathematics is ideology free. Unfortunately this opinion dominates between mathematics teachers recently too. Since 1990 - start of the political changes in Hungary - the ideological subjects were banished from the universities and the number of the pedagogical resp. psychological lessons decreased in a great manner. One mathematics secondary school teacher student has altogether only 4-6 lessons one semester long from Pedagogy and Psychology. Unfortunately the Didactics of Mathematics is separated from the Pedagogy and Psychology at the universities. The Didactics of Mathematics Group at the Eötvös Lorand University for example subordinated to the mathematical departments. There is no possible to obtain a PhD degree from Didactics of Mathematics in Hungary.

2. Lack of the international scientific literature. To get acquinted with the list of the rich international literature in the domain attitude, belief, motivation - presented by G. Törner at MAVI-3 (Törner & Pehkonen 1996) - we must establish, the lack of available current literature influences in a great manner the niveau and the significance of the works produced in Hungary.

Because of financial difficulties there is no money for example at the Eötvös Lorand University to order international journals and basic books. Since 2 years my department did not order any book , no journals, neither international nor Hungarian. If Hungary really wants to join the EU, the government needs to change the recent situation and must give more money for the education.

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Mathematics Versus Computer Science:
Teachers' views on teacher roles and the relations of both subjects

Peter Berger
University of Duisburg

Background

Subjects of my study are the computer science worldviews of teachers – strictly speaking, the investigation of the hypothesis that, by analogy with mathematical worldviews, there are also specific computer science beliefs to be found among German mathematics and computer science teachers. In this context, possible crossing-over effects between mathematical beliefs and computer science beliefs are of particular interest.

The investigation is mainly based on qualitative methods. The empirical material derives from 30 in-depth video taped interviews, made with teachers at secondary schools and comprehensive schools in North Rhine-Westphalia, one of the 16 federal states of Germany. A more detailed description of the study, along with a survey of some results, has been given in the proceedings of the first and second MAVI workshops (see Berger 1995, 1996).

In Germany, computer science is established as an independent school subject in its own right, far from being a mere computer course. Its main topics cover a wide range from solving practical problems by means of the computer, the theory of algorithms (including even efficiency assessments), up to formal languages and theoretical machine concepts. As a science-propaedeutical school subject in the classical sense of the word, it is essentially taught at schools preparing for university education, mainly (grades 9 to 13) at secondary schools and to a lesser degree at comprehensive schools. German computer science teachers for the most part started out as mathematics teachers, extending their qualifications in university studies or thorough in-service trainings in computer science. Not least for that reason, the historical and substantial connections between computer science and mathematics are present at German computer science classes.

In this paper, I focus on some of the above-mentioned crossing-over aspects: Regarding the special group of maths teachers who also teach computer science, it should be informative to gain an insight into their beliefs of the specific teacher roles and the relationships of both subjects. These beliefs we may understand as mathematical beliefs of insiders from an outside point of view.

Innovation vs. tradition: the roles of a good computer science teacher

As one result of the study it turned out, that the interviewed teachers actually do not see computer science predominantly as a science of the computer. So it would be more suitable to use the German term ›Informatik‹ (like French and Italian) instead of ›computer science‹. However, for reasons of convenience we will keep to the English term even at translated quotations.

Within the interview context concerning the views on a good teaching of computer science there had been the question »Are you a different type of teacher at computer science classes (compared to maths or other subjects)?«. If we standardize the spontaneous individual answers given in the interviews, we get the result shown in Diagram 1. By far the most teachers see their role at computer science classes different to that at their other subjects, especially to mathematics.

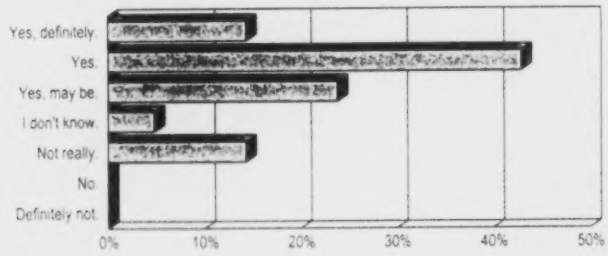

Diagram 1. Are you a different type of teacher at computer science classes?

How do the interview partners in detail describe their teacher roles? If we, from the total of about 200'000 words of all interviews, extract each ›meaning unit‹ (statement or term) making a contribution to the description of a teacher role, we find a clear grouping of nine main roles. All roles were assigned to a good computer science teacher. The teachers seem not to be interested in characterizing bad teaching. Some concentrate on one or two aspects, the most name several different roles. In detail, they refer to the roles of a ...

lecturer: »giving a lecture; is imparting knowledge; carrying knowledge over to students; says what things are like;«

distributor of marks: »makes assessments of students' achievements; gives marks;«

mediator: »trouble-shooter; mediator of conflicts between students;«

problem designer: »presenting problems; stimulator; bringing up problems; investing much time to find problems, in co-operation with students;«

service man: »provides the technical environment; has to take care that it works; net administrator;«
teamwork manager: »planner of projects and teamwork; helping students to make a product; project manager; organizer of the working process, planned along with or by the students; moderator of teamwork;«
member of a team: »after overcoming the first difficulties, I see myself rather as a member of different teams;«
consultant, coach: »I'd like best to be a mere ›answerer of questions‹; provider of informations; like a coach; walking around or just sitting down and the students will come to ask; playing the part of a consultant, not of a ›pusher‹; an expert, being at the students' disposal, but keeping out of the way;«
chairman: »chairman, moderator of discussions.«

Some of these roles (lecturer, distributor of marks, mediator) are traditional teacher roles, while some are distinctly innovative (teamwork manager, consultant, coach, chairman); some could be of both types (problem designer, service man). Diagram 2 shows how many teachers did name each particular role. It is remarkable that there are more innovative roles than traditional ones and, moreover, that innovative roles are significantly more often mentioned. Only at the third place we find the first traditional role of a lecturer. The two most frequently named roles (teamwork manager and consultant, coach) are innovative; more than 75% named at least one of these both roles. Only three teachers did not refer to an innovative role at all.

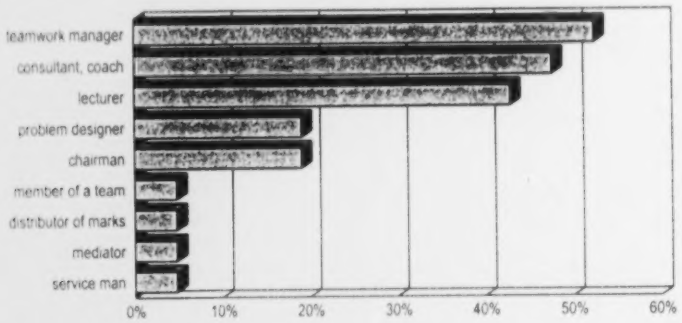

Diagram 2. Roles of a good computer science teacher.

If we count the frequencies of the teacher roles not based on the total of teachers, but on the total of namings, the outcome is as shown in Diagram 3. In the spectrum of teacher roles a dominant part of 62% of all namings refer to innovative roles.

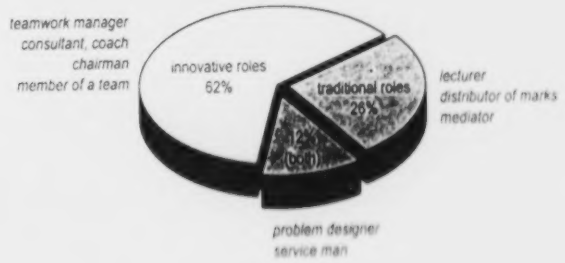

Diagram 3. Innovative vs. traditional teacher roles.

Computer science vs. mathematics: contrasting views and attitudes

This orientation toward innovation, as a thorough analysis of the interview statements reveals, may not be understood as a general attitude applying also to the teachers' other subjects. It is rather close connected with some specific views on computer science, which is seen in a remarkable contrast to mathematics. In the following, illustration of this will be given by some characteristic quotations referring to three aspects: the educational essentials of mathematics and computer science, the styles of teaching and learning, and the ›evolutionary status‹ of both subjects. The interview questions did not explicitly ask about teaching mathematics. But an average of about 8% of all interview statements are comments on mathematics, brought up by the teachers of their own accord.

The following quotations (from 10 different teachers) manifest a general difference in the views on mathematics and computer science with regard to *educational essentials*. While mathematics is seen oriented towards theory and formalism, computer science is characterized as practical, concrete, interdisciplinary, oriented toward applications, projects, and problem solving:

- »To me, the problem with maths is, that the questions are normally not as practical as they are at computer science. At maths many problems do not come to that point of everyday life. That's more distant.«
- »Of course, there is theory at computer science, too. But here even theory is somehow different to maths. At computer science things simply get more relaxed.«
- »A good project for maths? Hard to find – what should it be like?«

- »Opportunities for doing projects are given in a much wider spectrum at computer science. The project method has more importance for computer science classes. Projects for maths? I'd like to see some examples.«
- »Maths is too dry. At computer science, what I can do with maths, is to try it out and take a look on it. Algorithms and so on.«
- »At maths, some students are very interested in proofs and very motivated. But much more students will always ask ›What can I do with it? Is that any use to me?‹ and so on. And I think, questions like these are perfectly justified. A reasonable teaching of computer science can't do without application – and neither a reasonable teaching of maths.«
- »Well, I feel the teaching of maths and computer science are totally different. Of course, I'd like to teach maths in a problem oriented way as well. But the problems are not as comprehensive and complex as they are at computer science.«
- »As I see it, computer science is oriented towards applications. And theory here is developed along with the process of solving problems. Perhaps a mathematician would say, that it's the same with mathematics. But to me computer science first of all is application – application for solving problems and to make practical things a lot easier.«
- »Interdisciplinary aspects are inevitably involved with computer science. For example, when doing simulations, I have mathematical or economical subjects, I let die a lake, or explode a nuclear power plant. There are many topics behind: mathematics, biology, social sciences, and politics.«
- »For no other school subject I see such a good chance of interdisciplinary work: I've got a practical problem, which I'm going to solve by means of computer science; I may use mathematics as well, but I have to take in account even a lot of totally different aspects. I think, it is a fantastic school subject to recreate those synopses that we have lost through a narrow specialist thinking. At mathematics, problems mostly are artificial – at computer science I actually can tackle real problems.«

The same dissimilarities appear, if we regard the teachers' views on *the styles of teaching and learning* at maths resp. computer science classes. Maths teaching is characterized as frontal, teacher-centred, and dogmatical, whereas computer science teaching is associated with keywords as teamwork, creative, active, co-operative, co-determined, as the following quotations (from 11 different teachers) show:

- »Teamwork is much more often done in computer science than in math classes, where most teaching comes from the blackboard. At computer science things are easier and more relaxed.«
- »There is a small chance to do it even at maths classes – but mainly at computer science you can students let manage things their own way.«
- »At computer science – more often than at maths – I am a moderator: Someone, who is initializing certain processes, which will run by themselves, then.«
- »Teaching maths is more frontal. Teaching computer science is oriented towards teamwork.«
- »Learning objectives never are simple at computer science. Almost all is aimed at problem solving, creative work, something new. Of course, that requires a different kind of teaching. Whereas at maths certain topics can be taught rather dogmatically.«
- »At maths we fall back on frontal teaching (class teaching). Other social forms, for example teamwork, can only be realized at computer science.«
- »At computer science, I mostly keep back myself. Maths lessons are much more teacher-centred.«
- »Computer science is taught considerably more teacher-centred. Which at maths I cannot realize and perhaps not even want to realize, because of the great amount of material that is to teach at maths. At computer science, I take much more the liberty to respond to students' problems.«
- »At computer science classes one should keep out as far as possible. At maths the teacher role is much more dominant and I need more class teaching. When carrying specialized knowledge over to the students, even at computer science I need that. But not when I get students to create ideas – that is just where they can get active, and that's missing at maths.«
- »I feel considerably more insecure at computer science. But on the other hand I feel more relaxed, because I think that I can have much more confidence in the students than I could have at maths.«
- »At computer science you rather are ›peer among his peers‹ and sometimes you have to take care of not being too close friends with your students: that might happen frequently at computer science, whereas at maths you still tend to a teacher-centred lecturing.«

As an attempt of understanding the contrasting views on both subjects, perhaps latent underlying also the beliefs of their colleagues, some teachers refer to the different *›evolutionary status‹* of mathematics resp. computer science. They assess mathematics as old, completed, forma-

lized, inflexible, and in contrast to this computer science as new, in state of flow, and open-ended:

- »At computer science one is engaged in developing things further, whereas at maths one sticks to didactical concepts, which have been used a thousand times.«
- »Computer science is more in state of flow than mathematics is: as a science, and much more in its effects on society and education.«
- »There is always some kind of flow, there is always something new that makes me work more for computer science than for maths, where all is like stiffness, inflexibility.«
- »The fact, that computer science is not yet that definitive, and still is changing and turned upside down, actually is a chance to try out new things. You know that the maths curriculum is not given by God, but you are always in temptation to stick to it and to say ›who cares, I simply carry on like this, so it's printed in the book‹.«

Conclusions

Looking for an interpretation of our findings, we may consider two basic approaches.

Hypothesis 1: Computer science teachers *a priori* are innovative. Only innovative teachers will choose computer science as a new subject and will be motivated to undergo the trouble of long time in-service trainings or university studies of this subject, as most German computer science teachers actually did. But if that interpretation would apply, we should ask, why the interview partners do not turn out to be innovative also at maths? And for the most part they do not.

Hypothesis 2: Computer science teachers *a posteriori* are innovative. Teaching computer science ›makes‹ a teacher innovative. The evaluation of the interview material, together with the analysis of the situation of a computer science teacher in Germany, reveal that this conjecture points into the correct direction. Computer science as a school subject is not yet supplied with a sound background of didactical orientation and specific teaching methods. In this situation teachers are forced – and free – to search suitable models, even outside traditional schooling. At present, we observe a change of paradigms in teaching computer science. The traditional paradigm *school* – being characterized by keywords as lesson, homework, classroom test, teaching, assessing, examining, educating etc. – is not yet replaced, but more and more parallelized and superposed by the new paradigm of *professional life* with the leading concepts of project, product, team, discussion, consulting, delegating, co-operating (cf. Figure 4).

This changing, however, only in particular cases followed from a process of intentional didactical innovation and purposeful development

of a new teaching style. For the most part it took place by adopting a ›ready-made‹ system of approved social patterns from a domain outside school. This domain, being the world of computer and software professionals, has in most teachers' (and pupils') views obtained the status of a model of acting. The relatedness of a school subject to a model from outside school is a specific feature of computer science worldviews. Even at computer science as an innovative school subject, innovation appears not to be achieved by the innovative teacher, who creates a new paradigm of teaching – but by a new paradigm, that makes a traditional teacher act in a more and more innovative style.

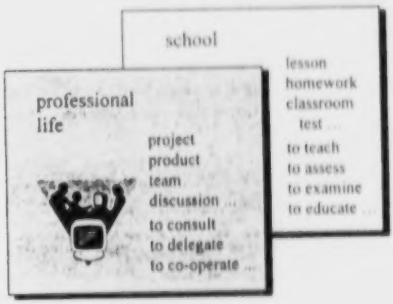

Figure 4. Two paradigms of teaching computer science.

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A theoretical framework for teachers' conceptions

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The teachers' conception of mathematics

The aim of this paper is to outline a model for studying the system of teachers' beliefs and conceptions. We base our considerations on the existing literature, including our works in the field.

The first problem arising concerns terminology, which differs according to the different authors and the different fields of research behind them (mathematics, psychology, education, mathematics education). Of course, there is also the additional difficulty of translating from the original languages of the researchers to English and viceversa. As an example I quote the case of Italy where the literal translation of the word *belief* (*credenza*) is less used in the common language than other words such as those corresponding in Italian to the terms *conception, idea, image, opinion, view*.

In the paper (Furinghetti, 1994) I have sketched a diagram illustrating what I mean when I use the terms image, belief, conception of mathematics. This paper was presented to a SCTP conference, where two other participants dealt with the same subject; confronting the approaches and the terminologies in their papers (Ponte, 1994; Vicentini, 1994) and in mine we can focus on some crucial points to better define my diagram. In (Ponte, 1994, p.169) it is assumed the view that «*knowledge* refers to a wide network of concepts, images, and intelligent abilities possessed by human beings. Beliefs and conceptions are regarded as part of knowledge. *Beliefs* are the incontrovertible personal "truths" held by everyone, deriving from experience or from fantasy, with a strong affective and evaluative component (Pajares, 1992)[1]. They state that something is either true or false, thus having a prepositional nature. *Conceptions*, are cognitive constructs that may be viewed as the underlying organizing frames of concepts. They are essentially metaphorical».

To investigate on the issues mentioned by João Ponte with reference to teachers we need to focus on *the mathematical knowledge for teaching*, which, according to the description in (Borko et alii, 1992) «consists of

[1] From the references in Ponte's article: Pajares, M. F.: 1992, 'Teachers' beliefs and educational research: cleaning up a messy construct', *Review of educational research*, v.62, 307-332.

two overlapping knowledge domains: subject matter knowledge and pedagogical matter knowledge». The expression «subject matter knowledge» is intended in a broad sense comprising the knowledge of technical aspects, notions and concepts (knowledge of the disciplinary contents), as well as the conception of the nature of the subject matter; «pedagogical matter knowledge» embraces knowledge of pedagogical theories, educational problems specific to the discipline, conception of the teaching of the discipline. For the purposes of the present paper we consider it useful to organize the elements at stake in the knowledge for teaching according to the diagrams in Figure 1 and Figure 2.

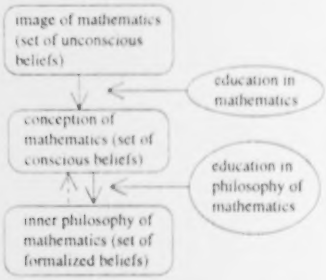
image of mathematics (set of unconscious beliefs)
education in mathematics
conception of mathematics (set of conscious beliefs)
education in philosophy of mathematics
inner philosophy of mathematics (set of formalized beliefs)

Figure 1. System of beliefs on mathematics

The figure 1 refers to the teachers' subject matter knowledge and focus on the conception of mathematics, presented at the three different status of *image*, *conception*, *inner philosophy*.

Image. In (Furinghetti, 1993) we have based our study on the hypothesis that any person having attended school has elaborated beliefs on mathematics. Usually these beliefs are unconscious; when conscious they are generic and fuzzy, relying on affective and evaluative component. The set of these beliefs constitutes the *image* of mathematics held by common people. We stress that the condition of having a school experience is essential, since we put the origin of the beliefs in it, distinguishing from the form of spontaneous mathematics as studied in ethnomathematics.

Conception. In 'professional mathematicians' (mathematics teachers, mathematics educators and researchers in mathematics) the beliefs on mathematics are focused and specific, relying on quantitative and rational components. Often they are unconscious, but the subject is able to reflect on them and to make explicit them through statements and examples, making them conscious. The set of these beliefs constitutes the *conception* of mathematics. When we mention the teachers' mathematics

conception we refer to this status. The two papers (Mura, 1993; Mura, 1995) are good examples of a way in which the research in conception can be carried out.

Inner philosophy. When the reflection on the beliefs reaches a level of formalization and may be confronted with the streams of philosophy of mathematics we have the *inner philosophy* of mathematics. The catalyst for this process of reflection can be a specific training in philosophy of mathematics and in foundational theories. In figure 1 we used a dotted arrow for underlying that from the existing studies it is not clear how the mathematics conception may be influenced by specific studies on philosophy of mathematics carried out by the subject.

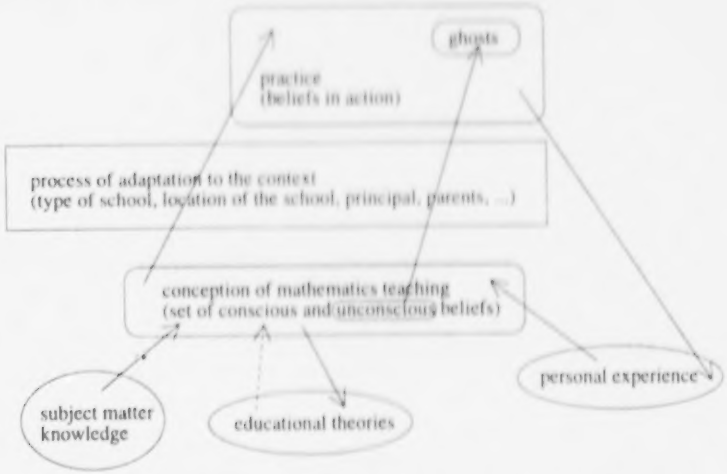

Figure 2. Teachers' beliefs on mathematics teaching

The teachers' conception of mathematics teaching

The figure 2 refers to the teachers' pedagogical matter knowledge and focus on the conception of mathematics teaching, considered as a product of the following factors: educational theories (in this expression we include pedagogical theories as well as educational problems specific to the discipline), subject matter knowledge, personal experience. The structure of the figure is aimed at emphasizing the process of passing from the conception of teaching to its realization in practice. We have enriched the dual scheme of subject matter knowledge and pedagogical matter knowledge introducing the process of adaptation to the context. This means we consider the teachers' *practical knowledge* defined in

(Elbaz, 1983) as the teachers' knowledge in carrying out their work. It «encompasses firsthand experience of students' learning styles, interests, needs, strengths and difficulties, and a repertoire of instructional techniques and classroom management skills. The teacher knows the social structure of the school and what it requires, of teachers and students, for survival and for success; she knows the community of which the school is a part, and has a sense of what it will and will not accept. This experiential knowledge is informed by the teachers' theoretical knowledge of subject matter, and of areas such as child development, learning and social theory. All of these kinds of knowledge, as integrated by the individual teacher in terms of personal value and beliefs and as oriented to her practical situation, will be referred to here a 'practical knowledge'» (ibidem, p.5). For the author the practical knowledge is «personal», «social», «experiential» and grows with the increasing of teachers' experience. In a certain sense the Figure 2 may be seen as a dynamic schematization of the practical knowledge.

In our model the conception of mathematics teaching is interpreted as the system of the *teachers' beliefs*. There are beliefs which are *conscious* (even non made explicit) and beliefs which are *unconscious*. This classification is fuzzy at the edges, nevertheless the importance of acknowledging the existence of unconscious beliefs is in the fact that they are present in classroom as *ghosts*. The *ghosts in classroom* are *unconscious beliefs in action*: they are the origin of inconsistency and disharmony in mathematics teaching.

To investigate on ghosts is not easy, since it is the same as to discover unconscious beliefs which are hidden for their nature itself. We have carried out investigations on the subject through the analysis of protocols, questionnaires, interviews. The issues were tackled from different point of view to single out contradictions and conflicts in the declared beliefs.

Also the comparison of teachers' declared beliefs and the beliefs in action allows to single out ghosts, but we have to take in mind that ghosts are not the only source of discrepancies between the teachers' declared beliefs and the beliefs in action: an important source is the process of adaptation to the context. Sometime this adaptation is so strong that the same teacher has complete different behaviours in classroom when the situation changes for some reasons (new school, new ages of students, different curricula in the school). I have examined the case of a teacher we interviewed during the preparation of the paper (Bottino & Furinghetti, 1996). When he was teaching in a school with a strong scientific orientation he used to introduce Euclidean geometry starting from classic texts (Hilbert). When he passed to a school which is aimed at preparing to the profession of hotel employers (mathematics is

a compulsory subject of the curriculum, but the students dislike it) he changed his teaching using recreational mathematics and mental calculation.

In the figure we have emphasized that the conception of mathematics teaching is the product of a teacher's personal elaboration relying on these main elements: subject matter knowledge, educational theories, personal experience. The subject matter knowledge has a strong influence in the shaping of the teacher' conception, since the teacher decides the strategies for teaching (choice of contents, degree of depth, weight to give to theory and to practice, ...) mainly starting from his/her knowledge of the disciplinary contents on which has built the conception of the discipline. The relationship with educational theories (we intend pedagogical as well educational problem specific to the discipline) is more problematic. In (Gadanidis, 1994) this relationship is considered as a source of *schezofrenia* for teachers when they compare their practice in school with the educational theories. We have assumed that the conception of mathematics teaching exists independently from knowing educational theories, nevertheless when a teacher becomes acquainted with educational theories can recognize that his/her conception of teaching is fitting to some theoretical issues. The dotted arrow in the figure means that on the ground of the existing studies we consider not clear if and how a specific training in educational theories can have an influence on the conception of mathematics teaching. We have only experience of indirect and mediated influences, e. g. behaviourism and the use of computer for instruction assisted by computer. The way teachers relate themselves to the educators has important consequences when teachers are asked by curriculum developers to accept innovation.

The conception of mathematics teaching has a strong reference to practice, since the personal experience creates beliefs on the students' learning and on the efficacy or usefulness of certain strategies. Successively these beliefs are checked in classroom and creates a new personal experience. This makes the diagram rather dynamic and dependent from the context. The 'degree of dynamism' depends on the 'degree of self-evaluation' on his/her school practice the teacher is able to activate.

Preliminary conclusions

Our theoretical framework for the teachers' conceptions can be used for studying some issues still under discussion. A problem we are considering in our studies is how teachers are aware of their conceptions and of the influence the conceptions have in their teaching. In retraining courses for teachers we have attempted to introduce a process of metacognition on the teachers' practical knowledge. This form of

metacognition appears particularly important for preparing teachers to accept curricular changes, especially when the proposed changes are in contrast with the existing beliefs and would need of a reshaping of teachers' conceptions. We have met some resistance by the teachers in carrying out this type of activity or, in other cases, a low interest, as if this kind of retraining would be perceived as a violation of the intimacy or as a completely unnecessary activity.

We are also studying how the conception of mathematics is generated and evolves: - at which school level is established, - how the mathematics studies at university influence it, - how it is possible to change it, - which are the sources of the beliefs.

Our model needs to be furtherly refined studying the nature of the other possible links existing between the various issues of the figures 1 and 2 (see the figure 3). To investigate on these links constitutes the ground for constructing a *philosophy of mathematics education*, in the sense illustrated in (Zheng, 1994).

Figure 3. Net of issues relating to philosophy of mathematics education

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Beliefs of Pupils and First-Year-Students about Mathematics Education

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Beliefs of Seven-Graders in Finland and Germany in 1991

On the basis of the idea of Erkki Pehkonen I made a questioning about beliefs concerning mathematics education between 12 to 14 years-old children. It was the same questionnaire Erkki Pehkonen used in several countries as already mentioned on this workshop. First results about the comparison of Finland and Germany we presented 1993 on the GDM-Tagung für Didaktik der Mathematik (see Graumann & Pehkonen 1993). The most important points I would like to refer to you:

The questionnaire was done in different schools in Bielefeld and in Helsinki with approximately 250 pupils in each city. Each question had a scale from 1 (total agreement) to 5 (total disagreement). The following picture shows the means (of all pupils of each city) for all 32 questions whereat we grouped all questions in five fields.

With comparing Finland and Germany we found the following 14 questions with significant difference (on the level of 0.1%):

- mechanical calculation (3)	Fin: 2.00 / D: 1.53
- to get always the right answer quickly (7)	Fin: 3.74 / D: 3.30
- there is always a procedure to get the result (10)	Fin: 3.07 / D: 2.57
- math. teaching should be understandable for all (11)	Fin: 1.85 / D: 1.47
- different topics are taught and learned separately (17)	Fin: 2.77 / D: 2.02
- the teacher explains every stage exactly (26)	Fin: 2.08 / D: 1.51
- there will be as much practice as possible (29)	Fin: 2.34 / D: 1.99
- it is necessary to understand as much as possible (30)	Fin: 2.01 / D: 1.42
- you sometimes make guesses and use trial an error (4)	Fin: 1.98 / D: 2.63
- strict discipline (8)	Fin: 1.51 / D: 2.40
- much has to be learned by memorizing (12)	Fin: 3.30 / D: 3.71
- learning mathematics requires a lot of effort (23)	Fin: 2.87 / D: 3.18
- use of calculator (14)	Fin: 2.04 / D: 2.98
- studying mathematics with practical benefits (19)	Fin: 1.52 / D: 2.02

If we try to interprete these differences we first can say (because of the last two items) that in Finland there is more emphesis on tasks with practical use. All other items may indicate that mathematics teaching in Germany is more traditional with learning contents and calculation being in the fore and not so much differentiation in method as it is

necessary in Finland where no outer differentiation with different schoolforms until 9. grade is in existance.

Beliefs of First-Year-Students

In this summer I had to teach First-Year-Students who study for teaching in primary school (1. to 4. grade). In Nordrhein-Westfalen all of them have to study also a little in mathematics and didactic of mathematics. In their first mathemtics lesson at university I gave to them a questionaire about their view on the coming study in mathematics and with the same 32 questions of the above named questionaire. I only changed the possibility of answering in that way that it was possible to answer first in respect to mathematics education they know from school ("is-state") and secondly in respekt to their wishes ("shall-state") because already in 1991 I thougt we have to differentiate between these both views. But at that time I wanted to use the same questionaire as use in Finland and other countries. But the following evaluation will show that it is necessary to make at least the named differentiation. I can not give a evaluation of all questions because of having not enough time and no help for the evaluation. I only took 6 questions among the above named 14 questions.

Before I will show the evaluation of these 6 questions I want to give you some general information about the test.group: Total number of returns 200 (83 % female, 16 % male, 1 % no statement); age 19-22: 77 % / age 23-30: 18 % / age 31-38: 3 % / no statement: 2 %; education in gymnasium: 87 % / already practiced a learned job : 12 % .

Now I will give the Evaluation of the 6 questions with most interest I thought. I always will show you the total number for each possible value and the mean for the is-state and shall-state. For better comparison I will refer the means of the above named questianaire in 1991. I also will give some numbers or/and a table of the distribution between is/shall-State and the scores.

Question # 3 (mechanical calculation)

	1	2	3	4	5	*no*
Is	55	104	22	8	0	*11*
Shall	64	91	27	5	2	*11*

Means: Is 1.91 Shall 1.89 / Test 1991: Fin 2.00 D 1.53

Number of test papers with the absolute value of the difference "is - shall" ≥ 2 is 5.

This question hardly doesn't show any interessting, exept that the mean of the students in Germany is much more nearer to the Finish

pupils than the mean of the German pupils. By the way, the number of test papers with the absolute value of the difference "is - shall" ≥ 2 (I call it a strict difference) seems to me a good indicator for comparing is-state and shall-state because one point difference often might be only light mood.

Question # 8 (strict discipline /concentration)

	1	2	3	4	5	*no*
Is	36	90	51	16	1	6
Shall	42	105	33	8	0	12

Means: Is 2.26 Shall 2.04 / Test 1991: Fin 1.51 D 2.40

Number of test papers with the absolute value of the difference "is - shall" ≥ 2 is 22 .

With this question there is already a remarkable difference between the means of is-state an shall-state an the number persons with strict difference between is-state and shall-state raised up to 11 %. Also we can see that the both means lie more or less in the middle of the Finnish and German means of 1991. Finally I would like to point out that the number for no-statements, especially the difference between is-state and shall-state, might be interesting. I think we can interpret this as expression for unsureness about the role of discipline or the meaning of the question.

Question # 11 (math. teaching should be understandable for all)

	1	2	3	4	5	*no*
Is	4	10	44	71	67	4
Shall	124	38	6	10	18	4

Means: Is 3.95 Shall 1.78 / Test 1991: Fin 1.85 D 1.47

Number of test papers with the absolute value of the difference "is - shall" ≥ 2 is 152 .

First we notice the high amount of the is-state whereas the shall-state lies near to the Finnish mean and not so far away from the German mean in 1991. But more important is the big number 152 (76 % !) of strict differences. Moreover out of the distribution comes that aleady nearly 25 % (49 persons) indicate the most possible difference with (is-state/shall-state) = (5/1) and 22 % indicate (is/shall) = (4/1). In addition the total number of persons with (is - shall) > 0 is 141. Only 4 persons are satisfied with the school-situation they have had and put down (is/shall) = (1/1) or (2/2). And only 11 persons want a change in

the other direction by putting down (is/shall) = (2/5), (1/4), (2/4) or (3/5).

I think the evaluation of this question shows very clearly that in such questionaires we have to differenciate between the is-state and the shall-state. Also it gives us a hint that at least in the higher secondary school in Bielefeld the mathematics teaching often is not understandable for all students.

Question # 26 (the teacher explains every stage exactly)

	1	2	3	4	5	*no*
Is	14	69	56	38	17	6
Shall	85	51	30	19	6	6

Means: Is 2.87 Shall 2.01 / Test 1991: Fin 2.08 D 1.51

Number of test papers with the absolute value of the difference "is - shall" ≥ 2 is 80 .

In this question there are not so much answers with strict difference but also a lot. The distinction is also not so much one-sided (there are 113 with the difference "is - shall" > 0 , 43 with is=shall and 35 with the difference "is - shall" < 0) means differ not that much. Interesting may be that the mean of the is-state is the biggest and the mean of the pupils in 1991 in Germany is the smalest one whereas the mean of the pupils in Finland and the mean of the shall-state of the Students are very similar.

Question # 19 (studying mathematics with practical benefits)

	1	2	3	4	5	*no*
Is	3	33	69	69	24	2
Shall	102	75	17	2	2	2

Means: Is 3.39 Shall 1.62 / Test 1991: Fin 1.52 D 2.02

Number of test papers with the absolute value of the difference "is - shall" ≥ 2 is 122 .

The distribution of the answers in detail is the following:

Is / Shall	1	**2**	**3**	**4**	**5**	*no*	**Sum Is**
1	2	1	-	-	-	-	3
2	19	9	3	1	1	-	33
3	32	30	6	0	1	-	69
4	35	27	6	1	-	-	69
5	14	8	2	-	-	-	24

no	-	-	-	-	-	2	2
Sum Shall	102	75	17	2	2	2	200

The evaluation of this question first conferms a big difference between is-state and shall-state. It also shows that nearly 87 % wish (59 % strictly wish) to study more mathematics for practical benefits, even most of those who have had studies of mathematics for practical benefits (18 %). Only 3 % of them have the feeling of to much mathematics with practical use and only 1 person whose study in school of mathematics with parctical benefits was neutral wishes still less. Because in respekt to the results of 1991 we can not be shure wether the children put down the is-state or the shall-state we can't make any clear interpretations.

Question # 4 (you sometimes make guesses and use trial an error)

Is / Shall	**1**	**2**	**3**	**4**	**5**	*no*	**Sum Is**
1	9	7	1	2	1	*1*	**21**
2	22	42	9	6	1	*3*	**83**
3	11	23	15	7	1	*2*	**56**
4	6	14	3	6	2	*0*	**38**
5	2	1	1	0	0	*1*	**17**
no	*0*	*0*	*0*	*0*	*0*	*1*	*6*
Sum Shall	**85**	**51**	**30**	**19**	**6**	**6**	**200**

Means: Is 2.58 Shall 2.19 / Test 1991: Fin 1.98 D 2.63

This question has a different distribution than the questions before. I leave it to you to find interesting points and possible interpretions.

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Feminine structures in mathematical beliefs and performances

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Below are presented some results from a Finnish research project considering secondary school pupils' mathematical beliefs and performances. They confirm the kind of findings obtained also in other studies of pupils' mathematical beliefs and gender-related differences in mathematics, and further point to some interesting interrelations between pupils' mathematical beliefs, their performances and some teacher- or school-related factors (see also Hannula, 1996; Malmivuori, 1996a, 1996b; Pehkonen, 1996). The subjects of the study consisted of 739 (363 girls, 376 boys) ninth-grade Finnish pupils from 50 schools over the country with 50 classes and the mathematics teachers of these classes.

The belief data was obtained from pupils' responses to the structured parts of a questionnaire measuring their views about the nature of mathematics and mathematical tasks, about mathematics learning, and about themselves as mathematics learners (Malmivuori, 1996a, 1996b). Pupils' mathematical achievement scores were measured through the national grade 9 examination, concentrating on mathematics at everyday situations with three different parts in it - i.e. mental arithmetics (10 tasks in 15 minutes), mental arithmetics with a calculator (10 tasks in 15 minutes) and word problems (5 tasks in 60 minutes), (Pehkonen, 1996). Variables on teacher factors were based on teachers' responses to a questionnaire with 28 (open and closed) items that covered teachers' backround information, teaching practices, mathematical beliefs, and evaluation methods (Pehkonen, 1996).

Differences between boys and girls

The results from mathematical test scores were consistent with the previous findings of gender-related differences in mathematics (e.g. Becker & Forsyth, 1994; Friedman, 1989; Kupari, 1983). Boys outscored girls on all the three parts of the test of the study. However, the difference in mental tasks with calculators was not statistically significant. The test score means for girls and boys and the results from the performed t-tests are given in Table 1.

Results from pupils' responses on mathematical views were also very similar to previous findings in Finland (e.g. Kupari, 1983).

Test	Girls		Boys		Unpaired t-value	Prob.
	Mean	Std. Dev.	Mean	Std. Dev.	(2-tail)	
Mental arith.	6,2	2,3	7,0	2,4	-4,744	,0001
Arith. with a calculator	4,5	2,3	4,7	2,3	-1,084	,2788
Problems	13,4	8,1	14,6	8,7	-1,944	,0522
Total	23,8	11,5	25,7	12,1	-2,218	,0269

Table 1. Differences between boys' and girls' test scores in mathematics.

Both girls and boys did not like mathematics very much. They found mathematics boring and difficult, but still viewed it as an important and useful school subject. In all, nine factors were constructed from pupils' responses to the rest of the structured items of the questionnaire, on the basis of the performed factor analyses (Hannula, 1996). Statistically very significant ($p<0.001$) gender-related differences were found in three of those factors. The two ones with largest gender differences are analysed here further. The first of these factors represented the constructed self-confidence measure on the questionnaire and the other factor was named as pupils' co-operation in mathematics. Self-confidence consisted of pupils' responses on statements like "I am not the type to do well in mathematics." or "I think I could learn more difficult mathematics.", with positive loudings refering to high self-confidence (see also Malmivuori, 1996a, 1996b; Malmivuori & Pehkonen, 1996). The items included in the factor of Co-operation are given below together with the related factor loadings. Note, that this factor includes also learning from mistakes.

"Co-operation as a way to learn mathematics":
You can learn mathematics by asking help from other pupils (0,73)
You can learn mathematics by thinking together with other pupils (0,66)
You can learn mathematics by making mistakes (0,66)
You can learn mathematics by asking as much as possible from your teacher during the lessons (0,43)

Factor scores for these two factors were calculated for all pupils. The statistically significant t-test values for the differences between girls' and boys' scores in these factors are presented in Table 2. Consistently with meny previous results (e.g. Bohlin, 1994; Fennema, 1989; Kupari, 1996; Leder, 1995; Reyes, 1984; see also Frost, Hyde & Fennema, 1994, with somewhat opposing results), but instead girls reflected more often than boys a tendency for co-operation in learning mathematics ($p < 0.001$).

Factor	Girls		Boys		Unpaired
	Mean	Std. Dev.	Mean	Std. Dev.	t Value
Self-confidence	-0,3	3,7	1,7	3,8	-6,54
Co-operation	2,7	2,0	1,9	2,3	5,00

Table 2. Factor scores for boys and girls. Both differences between sexes are statistically significant at the 0.1% confidence level.

Interrelations within classrooms

Correlations between pupils' self-confidence or co-operativeness, and their total test scores were calculated for boys and girls separately. Statistically significant correlations were found only between pupils' test scores and their self-confidence in mathematics ($p < 0.001$), where the positive correlation between total test scores and self-confidence was slightly stronger among boys than among girls. Other correlations were very small. In order to examine these interrelations and gender-related differences also at classroom level, mean scores of the three variables were computed for boy-groups and girl-groups within each class of the study. Below are presented the obtained correlations both at individual level and at classroom level (i.e. with the means of the scores) between these variables (Figure 3).

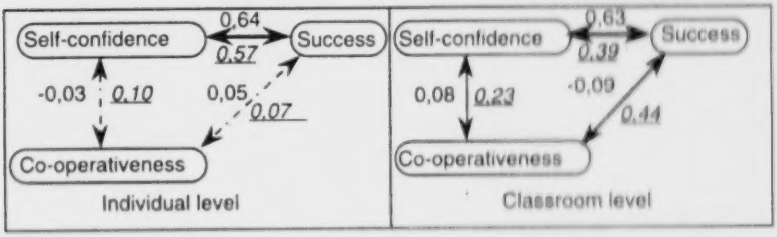

Figure 3. Correlations (separately for boys and *girls*) between self-confidence, co-operativeness and success on individual and classroom levels.

Some interesting correlations emerged at classroom level, that could not be found at individual level. At individual level there was no significant correlation between pupils' mathematics test scores and their co-operativeness, but correlations at classroom level displayed fairly strong positive connection between girls' test scores and co-operativeness ($p < 0.01$). This classroom-level interrelation was suggested to be connected to two different phenomena. First, it may indicate that high preference for co-operation within a group will improve especially girls'

possibilities to succeed well in mathematics. And again, when there are several succesful girls within the same class, there is also more chance that these girls will co-operate. This positive correlation was even slightly stronger than the positive correlation between self-confidence and test scores or between self-confidence and co-operation within girl-groups.

The strongest correlation within boy-groups could be found between means of self-confidence scores and mathematics test scores. Similarly as at individual level, the correlations between boy-groups' self-confidence and co-operation, and between their co-operation and test scores were very small, indicating rather low significance of co-operativeness for boys' levels of success in mathematics. Instead, there was a significant relation between boys' means of confidence and means of test scores. This strengthened the central and rather independent role of confidence levels for boys' performances (see also Malmivuori, 1996a; 1996b). The gender-related differences in girl- and boy-groups' self-confidence levels and in the correlations between self-confidence and success in mathematics are illustrated also in Figure 4.

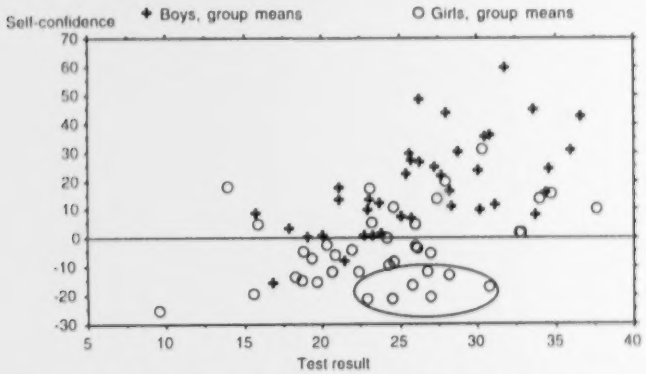

Figure 4. Self-confidence of girls and boys in different classes as a function of groups' test results (the classroom level). Seven low-confidence groups of girls are encircled.

In Figure 4 it is given the average of girls' and boys' mathematics self-confidence levels as a function of means of mathematics test scores for each class. As can be seen from the plotted values, girls had clearly lower confidence than boys in their abilities to learn and to do mathematics also at classroom level. Further, the positive relation between boy-groups' self-confidence and their test results is more apparent than that

of girl-groups'. This means that girl-groups with similar test (mean) scores are more likely than boy-groups unconfident of their mathematical abilities, and again girl-groups with consistent negative beliefs about own mathematical abilities may considerably differ in their mathematics performance levels. Especially interesting in this are the middle achieving girl-groups, that seem to express very low self-confidence, regardless of their average or above average test scores in mathematics. Seven of this kind of girl-groups are encircled in Figure 4.

Connections between pupils' beliefs and teacher factors

In order to consider the effects of some contextual factors in mathematics learning, correlations were calculated between the variables obtained from teachers' responses to the teacher questionnaire, and pupils' self-confidence, co-operation and test scores respectively. In Table 6 it is presented correlations between the self-confidence levels of boy- or girl-groups' and the teacher variables for these groups. All the statistically significant ($p < 0.01$) differences between girls' and boys' correlations (with teacher variables) are represented in the table. The same correlations calculated for pupils' test scores and co-operativeness in mathematics resulted in rather consistent findings with those given in Table 3 for pupils' self-confidence in mathematics.

Teacher variable	Correlation with self-confidence		
	boys	girls	Difference (stat. sign.)
Use of textbook's teacher manual for planning	-0,14 (-)	0,30 (-)	**
Use of school-made material for planning	-0,15 (-)	0,52 (***)	***
Use of school-made material for teaching	-0,06 (-)	0,52 (***)	***
Teacher feels need for research problems that enlighten the structure of mathematics	0,22 (-)	-0,22 (-)	**
How often one uses following working methods:			
Exercises in small groups	0,23 (-)	0,63 (***)	**
Solving problems in pairs or small groups	-0,17 (-)	0,40 (*)	**
Co-operative learning	0,07 (-)	0,46 (**)	**
Changes in teaching in recent years	-0,14 (-)	0,43 (**)	***
Mathematics tests have changed	0,16 (-)	-0,26 (-)	**

Table 3. Correlations between some teacher variables and the self-confidence levels of girl-groups and boy-groups. 'Difference' refers to the difference of correlations between boys and girls. All the teacher variables with statistically significant differences in correlations (at 1% level) are included in the table.

Very surprising was the result, that many of the teacher variables correlated only with the beliefs or successes of either of the two groups (boy- or girl-groups). As can be seen from the correlations, most often

this concerned girl-groups. The most (statistically) significant correlations were the positive correlations between girl-groups' self-confidence and their teachers' use of school-made material for planning and teaching, or of drills in small groups as working methods. Positive correlations were found also between girl-groups' self-confidence and their teachers' emphasis for co-operative learning and for use of pair or small group problem solving in teaching. Recent changes in teachers' teaching was also positively related with girl-groups' self-confidence levels, but instead the number of working years of the teachers' or the sex of the teachers' did not have any correlation with the self-confidence levels of either of the groups'.

A stepwise regression analysis was performed in order to find some examples of possible causal effects between teacher variables and girl-groups' levels of self-confidence in mathematics. Girl-groups' means of test scores together with four teacher variables (the best predictors) explained over 60 % of the variation in self-confidence levels between girl-groups, from which the four teacher variables explained the most variance (almost 60 %). The best two predictors were teachers' use of school-made material for planning and their use of drills in small groups. The third predictor - girl-groups' means of mathematics test scores - explained alone about 16 % of the variance in the self-confidence means of girl-groups'. The last two chosen predictors represented teachers' recent changes in their teaching and their views of mathematics as a process. The results of the performed regression analysis are given in Table 4.

Variable:	Coefficient:	Std. Err.:	Std. Coeff.:	F to Remove:
Teacher values also the prosess-nature of mathematics	0,384	0,194	0,196	3,926
Use of school-made material for planning	9,123	2,058	0,428	19,652
Use of working methods:				
Exercises in small groups	6,435	2,233	0,335	8,307
Changes in teaching in recent years	3,879	2,022	0,214	3,68
Test results of girls	0,65	0,224	0,284	8,408

$R^2 = 0.64$

Table 4. A regression analysis for girl-groups' self-confidence in mathematics and some teacher variables.

Epilogue

In the presented results for studying ninth-grade girls' and boys' mathematical beliefs, clear gender-differences were found in pupils' mathematical performances, in their confidence in learning and doing well in mathematics, and in their views of co-operativeness in learning mathematics. Also there was evidence that the influential aspects included in girls' constructive processes and mathematics learning in classroom context may differ from those features operating in boys' learning of mathematics. Here the studied impacts were found to be related to mathematics teachers' activity and especially to things as teachers' emphasis for co-operation in learning groups. The characteristics involved in the co-operative type of work in classrooms seemed to play an important role in girls' successes and confidence in mathematics, but not in boys' learning. As with girls, also boys' mathematics performances were highly positively related to their beliefs about own mathematical abilities, but the studied beliefs or performances of these boys could not be directly connected to their teachers' actions or to their own co-operativeness, as was the case with girls at classroom level.

Behind the studied variables and relations may be found a key to the explanations for girls' generally lower confidence in their mathematical abilities than that of boys', as well as to the possible ways of increasing girls' levels of confidence in mathematics. These factors could be traced back to the learning processes and environmental features operating in mathematics learning situations, that constitute the framework for the appearance of the different experiences and lives of girls' and boys' in and outside classrooms (see e.g. Bem, 1993; Gilligan, 1982; Leder, 1995). As the results above show, much responsibility for these features may be assigned to mathematics teachers and their actions, at least in the case of girls' learning of mathematics. Moreover, the kind of teacher variables considered here can be directly connected to the prevalent characteristics and processes of schools (e.g. factors reflected in the amounts of teachers' use of school-made material). Thus teachers actions may not arise only from their personal views, characteristics or experiences as mathematics teachers, but also from the features and lives of schools.

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Development of pupils´ mathematical beliefs: a description of a research project

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Summary

Below is presented a framework for a research project on the development of Finnish lower secondary school pupils´ mathematical beliefs. The project is aimed at clarifying the ways that pupils´ mathematical belief structures are further constructed in different mathematics learning contexts and with different mathematical experiences. A special interest is given for the factors that are connected to the development of pupils´ self-regulation in mathematics learning situations. The project will consider these developmental aspects through pupils´ mathematical meanings, motivation, emotions, learning behaviors, and through their mathematics achievements. Attention will be paid also to the gender-related differences in the development of mathematical belief constructions and mathematical performances of pupils´.

A research project was designed to consider how pupils´ mathematical beliefs and belief systems develop within different classroom contexts at lower secondary school level for the period of three years. The project will get started in the autumn 1996 with seventh-grade pupils from schools in Helsinki and its neighbourhood, and will be going on with the same pupils through the grades eight and nine up to the spring 1999. The project directed by Dr. Erkki Pehkonen will be performed in the Department of Teacher Education at the University of Helsinki, and be financially supported by the Finnish Academy. It includes two fulltime researchers (Mr. Markku Hannula and Ms. Marja-Liisa Malmivuori) for three years.

1. The framework for the whole project

The purpose of the project is to find out how pupils mathematical beliefs and belief systems are developed through lower secondary school level (grades 7–9). Especially, we are interested in what way pupils´ mathematical experiences and contextual factors are related to these constructive processes, and how pupils´ beliefs direct their learning and behavior. The main idea of the project is to concentrate on pupils´ mathematical belief constructions, their reflections and behavior, taking place within the learning contexts constituted by their personal

experiences with mathematics, their mathematics teachers, and their school environments. A special attention is given to the ways in which pupils approach mathematics or mathematics learning situations. These approaches will be considered through four different aspects: through pupils' mathematical meanings, through their feelings with mathematics, through their motivation in learning mathematics, and through the functional aspect in their learning. The first category represents pupils' conceptions and significances attached to mathematics and its learning (A), the second category relates to their affective responses and experiences with mathematics (B), the third category deals with pupils' short and long term orientational basis for mathematics learning (C), and the fourth category involve pupils' real actions and performances in mathematics learning situations (D).

Realization of research

Many kind of both quantitative and qualitative research methods will be at use during the project. And we will also be open to develop the research problems as the project proceeds. In the realization of the project, we will use mainly two different approaches. The quantitative part of the project (Malmivuori) which is characterized as a macro-level approach to the research questions is based on the considerations of the beliefs, reflections and performances of all the participating pupils.

The qualitative part of the project will concentrate on the developmental aspects of mathematical beliefs within the groups of pupils from two schools (a micro-level approach). In one school, the teachers have a more constructivist type of approach to mathematics teaching and learning than in average. This approach is viewed to include aspects as pupil-centered teaching, support for pupils' own activity and independence, whole group and small group discussions, small group work, creativity at use, and use of various kinds of hands-on materials. In the other school, the development of mathematical beliefs will be additionally actively affected and promoted by the teacher (Hannula), who at the same time acts both as a mathematics teacher and as a researcher. A special emphasis is given also to the kind of effects that are viewed to contribute especially to girls' experiences and beliefs in learning mathematics, particularly their self-confidence in mathematics. The structure of the whole project is sketched in Figure 1.

Test persons

The subjects of the project consist of lower secondary school pupils from 17 schools in Helsinki and its neighbourhood. Fifteen of these schools are selected at random, and two study groups of pupils are systematically chosen. The two additional schools are selected on the

basis of their special mathematics learning contexts involved in the schools and/or created by their mathematics teachers, i.e. a constructivist type of approach to mathematics teaching and learning. The schools and mathematics teachers in the project form the basis for backround information or of the contextual features that essentially may affect the constitution of pupils' mathematical beliefs and experiences within classrooms.

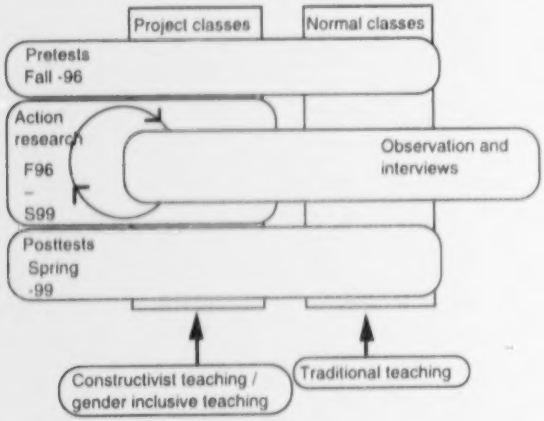

Figure 1. The framework for the project considering the development of pupils' mathematical beliefs.

2. The macro level approach to pupils' mathematical beliefs

The quantitative part of the study deals with the developmental aspects of lower secondary school pupils' mathematical beliefs rather at school and group levels than as changes in individual learning processes. This means that mathematical beliefs and performances will be considered against various pupil-related characteristics and factors attached to school or classroom contexts. A special interest is given for the differences in girls' and boys' constructions, but also the considerations are to uncover the kind of developmental patterns that will essentially combine the variation in pupils' mathematical beliefs, experiences and performances. This part of the study is further designed to make connections between pupils' belief structures and their mathematics learning environments. These will be based on the information obtained from the questionnaires for mathematics teachers

and schools of the study (i.e. for the headmaster of the schools) about teachers' mathematical views, as well as about aspects in their teaching of mathematics and in the educational conditions or instructional arrangements within each classroom and school.

Most of the data in the macro level part of the study will be gathered through self-report questionnaires for pupils, measuring their beliefs about mathematics, mathematics learning and teaching, themselves or their actions in mathematics learning situations, and their different affectively toned responses and experiences with mathematics. Same questionnaires will be at use both at the beginning and in the end of the secondary school years, but pupils' mathematics achievement levels will be checked then by separate mathematics tests. During the three school years there will be some more questionnaires, projective tests and/or e.g. check-lists measuring pupils' responses or actions in specific mathematics learning situations. These together with performed interviews and observations of pupils will be used to complete the obtained results from questionnaires and mathematics tests.

The leading idea for considering the developmental aspects in pupils' mathematical belief structures and performances in this part of the study is represented by self-regulated mathematics learning. This look on self-regulative actions as central and expedient processes for pupils' promotive mathematical belief structures and experiences is strongly based on the view of pupils as active participants in and creative constructors of their own learning in socio-cultural mathematics learning environments – as is suggested in the recent constructivist view of learning. In the framework of self-regulation it is referred further to aspects as pupils' systematic use of metacognitive, motivational and/or behavioral learning strategies, and to their monitoring, direction, and regulation of own learning processes (e.g. Zimmerman, 1990). These actions are further viewed to be linked to the variation in pupils' motivational determinants and processes in learning mathematics in which their forms of self-actualization are regarded here as the most central and far-reaching experiences in pupils' mathematical lives (see also Malmivuori, 1996b).

The picture given (Figure 2) illustrates this framework and some of the essential factors for considering the development of pupils' mathematical beliefs, experiences and performances in the macro level part of the study. According to this view chosen, pupils' mathematical beliefs are seen to develop through their personal learning experiences with mathematics that in turn take place within the socio-cultural learning environment of a classroom and of a school. The core of these personal learning experiences is again viewed to be represented by different forms of pupils' self-regulation in mathematics. These constitute the most important scene for pupils' mathematical beliefs, responses and actions to be in

action, reflected and developed. Through these processes pupils will further construct their beliefs about self, about mathematics, and about mathematics learning and teaching, as well as their responses toward mathematics and its learning. Effects of these constructive processes will then be reflected in pupils' mathematics achievements or their choices of mathematics and mathematics courses.

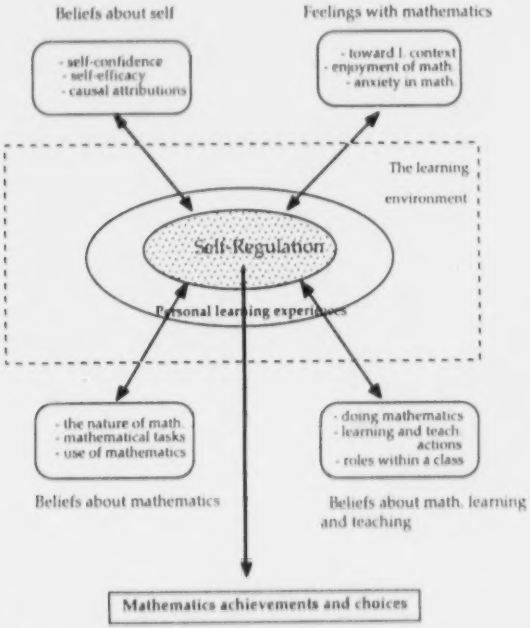

Figure 2. The view to the development of pupils' mathematical beliefs and performances.

3. The micro level approach to pupils' mathematical beliefs

The micro level approach to the development of pupils' mathematical beliefs is designed to observe and examine pupils' learning processes and experiences in different mathematics learning situations more accurately, as well as possibilities to influence them. Individual pupils and their belief structures represent then the targets of the study. This approach will afford an opportunity to find out the individual develop-

mental processes behind each pupil's mathematical belief constructions, and also to look at the ways that pupils' belief structures are related to their unique experiences and actions in mathematics learning situation. In the action research part of the micro level approach, the researcher who is at the same time the teacher is viewed to actively participate and contribute to his pupils' individual processes and experiences with mathematics. A special emphasis will be given for girls' developmental processes.

In most achievement tests boys have outscored girls in mathematics. Under 11 ears of age no significant differences have been found, but by the age of 15 a gender gap is seen in most tests (e.g. Friedman, 1989; Becker & Forsyth, 1994; see also Hannula & Malmivuori, in this proceedings). The width of the gap is not large and depends on the test used (Leder, 1992) and the country in question (Hanna, 1994). Moreover these differences have been decreasing over the years (Hyde, 1981; Rosenthal & Rubin, 1982; Friedman, 1989). The gender differences in mathematical beliefs are generally larger than in achievements. This apply especially to middle achieving pupils (Frost et al., 1994).

The most clear gender difference is found in pupils' confidence in mathematics, which has been reported several times (see Hannula & Malmivuori, in this proceedings). This difference seems to emerge first in confidence levels and later in achievement tests, making girls doubt their own mathematical abilities (Licht & Dweck, 1987). It is obvious that the biological sex of a child can not be a direct reason for these gender differences in mathematics. In this we need to turn to the social aspects of sex and to the concept of gender (on 'gender' see Bem, 1993).

In this project we see gender as a construction. The construction of a childs' gender identity is based on his/hers life experiences in home, with peers, through media and in school. By observation and interviews we try to learn how these experiences determine the construction of gender, having a special interest in its relation to mathematics. As girls have lower self-confidence in mathematics, we suppose that many of them see femininity and success in mathematics somehow exlusive. In this project we try to act so that every child would find mathematics compatible with his/hers gender identity. As we have only very limited control over the construction of gender identity, we focus on the pupils beliefs in mathematics.

4. Connections to our previous research

From the beginning of the ninties, we have been interested in and investigated pupils' mathematical beliefs. Our earlier results gave us the

rationale for the on-going research project. Therefore, the earlier results which are grouped into three categories (pupils´ mathematical beliefs, differences between girls and boys, contextual factors and beliefs) will be dealt with here briefly.

Pupils´ mathematical beliefs

Results from our previous studies on Finnish secondary school pupils´ mathematical beliefs and performances have been quite consistent with those findings acquired abroad (e.g. Frank, 1988; Garofalo, 1989; McLeod, 1989; Schoenfeld, 1989). Both seven-graders and ninth-graders tend to view mathematics as an important and highly useful school subject. Still they do not like mathematics very much and seem to classify mathematics more as boring and demanding than interesting or fun. (Hannula & Malmivuori in this proceedings; Pehkonen, 1992; 1996) There are also similarities between the traditional view of mathematics and secondary school pupils´ beliefs about the nature of mathematics and its learning and teaching. For example, pupils emphasize calculation, practicing and rigorous working methods, and they also require their teacher to give help and directions whenever needed (Pehkonen, 1992).

Some important relations has been confirmed also between pupils´ mathematics performances and their mathematical belief structures. Consistently with foreign results, pupils´ confidence in their learning of mathematics was found highly positively related to their mathematics test scores and operated a significant predictor of the scores (Malmivuori, 1996a; Malmivuori & Pehkonen, 1996). Also constructions as pupils´ beliefs about mathematics usefulness, their view of mathematics learning as based on own effort or on co-operation predicted positively their mathematics test scores. Again a belief on a fixed mathematical ability, on correct and quick solving processes and on the importance of innate mathematical ability related negatively to pupils´ scores. (Malmivuori, 1996a) Confidence in mathematics appeared to be further significantly and positively intertwined with pupils´ preferences for own effort and self-regulation in doing mathematics as as well with their views of mathematics as useful (Malmivuori, 1996b).

Differences between girls and boys

Differences has been found between girls´ and boys´ mathematics achievements in the performed studies. Generally boys tend to outscore girls in mathematics tests but girls´ grades in mathematics may be better than boys´ (Hannula & Malmivuori in this proceedings; Pehkonen, 1992; 1996). Boys´ better performances show up especially in mathematics tests involving open problems or independent thinking (Pehkonen, 1992;

1996). More significant gender differences however appear in some of girls' and boys' mathematical beliefs. This apply particularly to pupils' levels of confidence in mathematics in the favor of boys, but again girls more than boys tend to reflect a constructivist type view of learning and teaching mathematics including e.g. co-operation, approval of mistakes in doing mathematics, lower emphasis to innate mathematical ability, and stressing of procedures instead of mathematical outcomes (Hannula & Malmivuori in this proceedings; Malmivuori, 1996a; 1996b; Pehkonen, 1992; 1996). Gender differences in these constructions will cause different mathematical experiences and are importantly linked to the variation in girls' and boys' motivational processes in learning mathematics (Malmivuori, 1996b), producing thus further differences in their mathematics achievements.

Contextual factors and beliefs

Results dealing with secondary school Finnish pupils' beliefs about some environmental or contextual factors have reflected the significant role of teacher in learning mathematics (Malmivuori, 1996b; Pehkonen & Tompa, 1994). This tendency appears especially in girls' mathematical views and preferences, boys concentrating also on things as learning tools or materials at use and on working methods in classrooms (Pehkonen, 1992; 1996). In the performed preliminary analyses on ninth-grade pupils' mathematical beliefs, several statistically significant positive relations appeared between girls' self-confidence, their views of co-operativeness or their mathematics test scores and things as their teachers' use of school-made material, use of co-operative methods in mathematics teaching, or the number of recent changes in teaching. These relations could not be found between the studied boy-groups and their teachers. Four of the most significant teacher variables explained almost 60 % of the total variation in girl-groups' self-confidence levels. Additional analyses at group level further revealed that even if (especially middle achieving) girl-groups reflected low confidence, their emphasis for co-operation in learning mathematics may have a more important impact on their successes at classroom level than among boys. (Hannula & Malmivuori in this proceedings)

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Mathematical beliefs of eight-graders: What is mathematics?

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The aim of this study is firstly to find out what kinds of mathematical beliefs the pupils have and secondly to examine the change in the beliefs during the lessons in three years. The study will be an action research, in which the researcher and the teacher are the same person. The test group is one of the groups the teacher teaches. The other pupils of the same grade are the control group.

This report is a part of a pilot study made in August 1995. 84 pupils in grade 8 in a secondary school had to fill in a questionnaire about their views of mathematics.The test group consist of 18 pupils. Here the test group is regrouped so that the pupils are all going either to a high school, a technical school or a college of technology. The test group has been interviewed in groups of 3 - 4 pupils..

The research problem in this report is, *what is mathematics*. This question has lead to the following new questions:
(a) What are the mathematical contents?
(b) What is the way of doing mathematics?
(c) What is the way of learning mathematics?

What is mathematics? - the mathematical contents

First in the questionnaire[2] there is an open question about mathematics. The pupils are asked to write what the word 'mathematics' brings to mind. What do they think mathematics is? What kind of things belong to mathematics?

The pupils have written 274 comments in total about mathematics. The conclusion of these is that mathematics is computation. They give some examples: addition, subtraction, multiplication, and division. Some say also fraction, power and percentage. Mainly mathematics is arithmetics to them. Only one pupil says that mathematics brings to mind geometry. 40 % of all the comments they have is computation. Computation forms 53 % of the comments of the test group while the percentage of the control group is 35 %.

The word mathematics brings to their minds also mathematical tools like a pencil, an eraser, a ruler, a pair of compasses, a mathematics

[2] The questionnaire used was developed by Ms Marja-Liisa Malmivuori for the purposes of the Kassel project (see e.g. Blum & al. 1992).

textbook and a notebook. (the testgroup 6 % and the others 10 %). Also the tests are almost 10 % of all the comments in both groups. Mathematical problem solving is mentioned by only very few (8 % and 3 %). Figures, expression and a system of coordinates together are less than 10 % (6 % and 2 %). Homework is mentioned in 8 % and 7 % of the comments. Pupils in the control group have written: difficult exercises, even terrible homework.

The test group has mentioned numbers, but it is unclear what they refer to. Grades are not mentioned at all. Some have mentioned teachers and a fish and a teddy-bear in the book of grade 1. The test group have on average 4 ideas. The number of ideas varies from one to 12. None of them is negative.

The control group has also lessons in mind and mostly their comments are negative. The teachers are mentioned more often by the control group than by the test group. Mathematics is also said to be important and tough. Some have used expressions like assuring the future, regularity, possibility to work in peace, a schooldesk and a chalk etc.

Most of these comments are cognitive. The affective side of mathematics is asked in the questionnaire. The pupils have statements of mathematics, which have to be (dis)approved of a scale ranging from +5 (total agreement) to -5 (total disagreement). They can also choose 0, if they do not know what to say.

Mathematics is:
1. important
2. necessary only at school
3. difficult
4. interesting
5. terrible
6. necessary after the school
7. fun
8. tough
9. boring
10. necessary only for some people
11. demanding

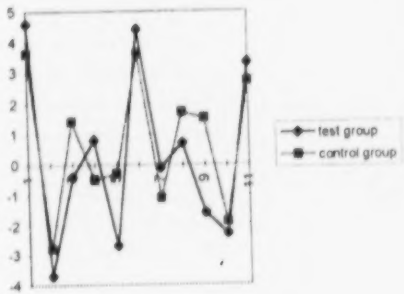

Figure 1. What are the contents?

When considering the results of the questionnaire, the statistics used are means and standard deviations for both groups of pupils, and the student t-test.

The pupils in the test group totally agree that mathematics is important and the pupils of the control group agree on it. Only two of 84

pupils disagree and a few pupils don't know it. Mathematics is necessary after the school, too, not only at school. The standard deviations are bigger than in the statement 1. Interesting are the affective statements: mathematics is difficult (3), interesting (4), terrible (5), fun (7), tough (8) and boring (9). The standard deviation of each of these statements is big.

When considering the answers of all pupils they agree that mathematics is demanding. They cannot say if mathematics is interesting or terrible. They slightly agree that mathematics is difficult, tough and boring, and not fun. Only some pupil say that mathematics is fun at school.

If we had a "mean-value" pupil in the test group he could not say if mathematics was difficult, interesting, terrible, fun or tough. He slightly disagree that mathematics is boring and agree that mathematics is demanding. One of the pupils in the test group totally agree that mathematics is difficult, interesting and demanding and totally disagree that it is terrible, tough, boring and fun. Another pupil in the same group totally agree that mathematics is demanding, interesting and fun and totally disagree that it is difficult, terrible and boring. In the test group or in any other group there are no "mean-value " pupils. They are individuals with their own values, opinions and experiences..

What is the way of doing mathematics?

In the questionnaire there are nine statements about the way of doing mathematics. The replies of both groups are rather equal. Statistically there is only one difference. It in the statement 2. The test group totally diagree that one should always get the right answer quickly, the control group only disagree it.

In the USA for example Frank (1988) had explored the question: "What do students believe about mathematics?" One of the beliefs is "Mathematics problem should be quickly solvable in just a few steps." "They believed that something was wrong either with themselves or with the problem itself if problem took "too long" (more than five to ten minutes) to solve." These pupils disagree the same statement. The interview of the test group confirm their answers. Many pupils say that when they have a problem to solve they can think of it half an hour, one hour or even two hours. Only few say

When you solve mathematical tasks
1. doing mistakes shows bad knowledge
2. you should always get the right answer quickly
3. you need much more than mechanical computation skill
4. only mathematically talented persons can solve the most tasks
5. there is usually only one right solution
6. you need to ponder a lot

7. small mistakes surely lead to a failure
8. the right answer is more important than the mathematical method
9. there is always a direct procedure that reliably leads to the right result

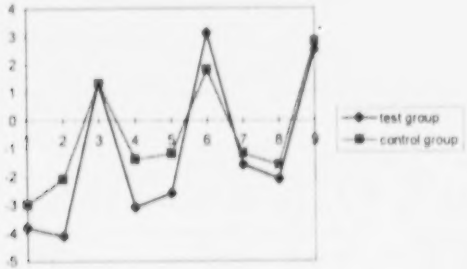

Figure 2. What is the way of doing mathematics?

that they leave the problem in five or ten minutes. Doing mistakes is not so bad. Somebody says that you can learn by doing mistakes. They believe that there is always a certain procedure that leads to the right answer, but there could be another, too. In order to get the result you have to ponder a lot. However you need not to be mathematically talented to solve the tasks. The mathematical method can be more important than the right answer.

What is the way of learning mathematics?

In the questionnaire there are 14 statements dealing with the way of learning mathematics. The mean values of both groups are nearly the same. The pupils disagree only on one statement, number 12, according to which you can learn mathematics only if you are talented enough. They all agree on the statement 9, when learning mathematics you have to do a lot for it yourself. The pupils in the test group think that it is important to practise the right procedures and to ponder things together with the other pupils. They agreed the statement that you can learn by listening to the teacher. The control group totally agree on the statement 2, learning by listening to the teacher. The control group think that it is better to learn when the teacher tells what to do while the test group emphasizes that in learning you have to ponder with other pupils, you have to ask them for help, you have to practise the right procedures and to do as much as possible for mathematics.

1. only if you always understand all the things that has been taught
2. by always listening carefully to what the teacher says
3. by doing mistakes
4. by asking other pupils for help
5. by doing exercises as much as possible
6. by reading the mathamatics textbook carefully
7. by thinking over things together with the other pupils
8. by asking the teacher as much as possible
9. by working hard for it
10. only, if the teacher teaches well
11. by thinking and doing the exercises alone
12. only, if you are talented enough in mathematics
13. by learning many things by heart
14. by practising the right procedures in tasks as much as possible

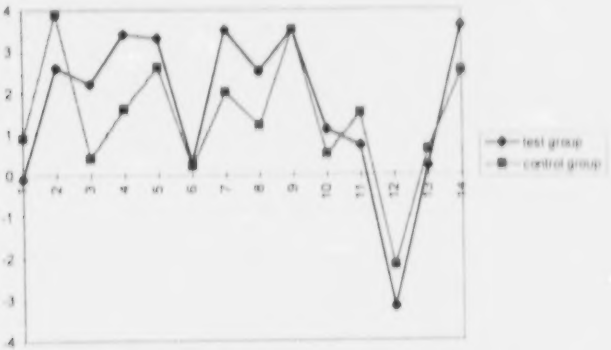

Figure 3. What is the way of learning mathematics?

What will be the continuation?

After two years learning it is time to give the pupils the same questionnaire again. Then it is possible to compare the opinions of the pupils. What do they then think about mathematics? Compairing the opinions of both groups and interwieving the pupils of the testgroup it could be possilbe to explain the differences between the opinions of the testgroup and the control group.

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Prospective Teachers' Math Views and Educational Memories

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Background

The study of the paper at hand is based among others on the theory of the development of teachers' beliefs about mathematics teaching by Alba Thompson (1991). The individual's *mathematical beliefs* are seen to be composed of his subjective implicit knowledge of mathematics and its teaching/learning. *Conceptions* are understood to be conscious beliefs. The beliefs - conscious and unconscious - can be seen as a *belief system*. As the individual's belief system is tangled with his knowledge system it is often quite difficult to distinguish between these two. (Pehkonen & Törner 1996; Lindgren 1996.) When the object of the belief system is mathematics or mathematics teaching/learning I use the term *view of mathematics*.

The world of beliefs, conceptions, and values is complex and dynamic. The research literature on teachers' mathematical beliefs shows the difficulty of identifying the "coloured" beliefs developed during previous school experiences. It seems evident that the spectrum of the variety of math teachers in the prospective teachers' educational history plays an immense role in the inception, development, and manifestation of the student's math view. Thus it is of great importance that teachers in pre- and in-service training be aware of their own beliefs about mathematics learning and teaching and that they are helped to construct new beliefs. (Kupari 1996.)

Foss and Kleinsasser conducted a profound study concerning the possibilities for changing the math views of prospective elementary school teachers. The researchers collected both quantitative and qualitative data from 22 student teachers. The target group was enrolled in a 16-week (three credits) long method mathematics course taught by an enthusiastic young teacher who frankly declared a constructivistic viewpoint on teaching and learning. Her philosophy was epitomised in concrete, active, interactive lessons and she directed her students to experience mathematics not just follow a textbook.

In addition to confronting the student with a Likert-scale questionnaire at the beginning and the end of the course the researchers made three interviews and two videorecordings with each student. They also gathered data by following the method lessons once a week and also

from essays and posters made during the course. Their results were saddening. They write: "In the main, the pre-service teachers' conceptions of mathematics remain constant during the time they spend in their mathematics method course ... Often unsure of how to define it, these participants perceive mathematics in relationship to their own mathematical experiences and rarely reference their mathematics method course " (Foss & Kleinsasser 1996, 434). It seems that the student teachers appear to ignore the general philosophical disposition of the course and rely on knowledge from the past. Students' practicum lessons also illustrate similar views. They place emphasis on practice, drill, and memorisation.

These findings are not encouraging. The researchers suggest among other things that in the practica, co-operating teachers, the method instructor, and the student teachers should meet for discussions of philosophies and instructional strategies. Pre-service teachers should learn to justify their actions through discussion and debate. (Foss & Kleinsasser 1996, 441.)

Objectives, methods and subjects of the study

My interest has been in the structure of beliefs and conception of teaching mathematics and the issue of the possibilities of changing student teachers' math views. My hypothesis is that change can be achieved, but that it is not enough only to give the students a method course emphasising a constructivist approach to teaching mathematics. The students need to *experience* that *mathematics* can be studied and learned in a constructivist manner. In my teaching of the basic courses of mathematics I want to give my students hands-on experiments with manipulatives and problem solving. In accordance with the theory of Watzlawick (see Lindgren 1995a) I use videofilms, aphorisms, metaphors, and games in order to influence students' "world images" through "right hemisphere strategies".

I gathered quantitative and qualitative data from the prospective elementary school teachers I am teaching (together 163 students) at the Department of Teacher Education, University of Tampere. I used a Likert-type belief inventory for a group of senior students and two cohorts of freshman students beginning their studies at the Department of teacher Education at the beginning of the autumn terms 1993 and 1994.

The first group of 72 freshman students completed the questionnaire twice - at the beginning and the end of the academic year. My intention is to follow this target group through the phase of teacher education. On the basis of a preliminary analysis of the data, and a math exam, 12

students were selected for closer follow-up. They have been interviewed, and lessons included in their second year teaching practicum have been videotaped (Lindgren 1995a). In the interviews attention was paid to the emotional memories of these students' math teachers. My intention is also to observe the mathematics teaching of those which can be reached and give their consent during the first year in the field. In this paper I concentrate presenting the results obtained from the interviews with these 12 student teachers.

Results

Procedure. Alba Thompson (1991) asserts that the development of teachers' concepts about teaching mathematics evolves in a manner which can be seen to be hierarchical in structure. The factor analyses of data from the combined group of 163 students gave me the hypothetical model of three partly overlapping levels. I call these: level OA (Open-Approach), level DG (Discussions and Games), and level RR (Rules and Routines). These levels refer to Thompson's levels 0, 1 and 2. The measures for the levels OA, DG, and RR were obtained as means from certain items. For these levels some attributes are the following (For a complete list of the items see Lindgren 1996.).

Open-Approach:
- to experience that the same result can be achieved in different ways.
- to encourage the students to find different strategies for solving problems, and to discuss these strategies.
- the use of concrete manipulatives
- to emphasise the importance of mathematical thinking.

Discussions and Games:
- the teacher should try to promote active class discussions
- the teacher must not emphasise individual work
- the teacher should let the students use many learning games
- the teacher should promote the pupil's ability to work with other pupils.

Rules and Routines
- to teach mathematical knowledge i.e. facts, rules, and statements
- it is most important that the students get the right answers
- it is most important that pupils practice extensively
- the pupils should learn to master basic calculation.
- the most important task for the teacher is to maintain good order in the class.

In the questionnaire each item was scaled from 1 to 5, where 5 meant full agreement with the proposal of the item. For each student the measures for the levels OA, DG, and RR were obtained by counting algebraically the means for the items that formed a certain level. The

values of these means were then standardised; the means for these levels for the whole target group (n=163) were zero and the standard deviations one. The values used were counted from the second measurement.

Findings regarding the levels. The frequency distribution of OA was arranged and the distributions of DG and RR were analysed for students with different values on level OA. For those students with low OA (n=23) and those with high OA (n=22) the distributions of the levels show clearly how the preferred teaching methods for these two groups are quite opposite. The low OA group appreciates rules and routines. For those with high OA there is no agreement about the best way to teach mathematics. There are candidates who have a high DG value and a low RR value, and others with high RR value and somewhat lower DG value. There are also student teachers who have high values on all of these three levels.

From these analyses and the interviews conducted I made the inference that the level DG can be seen as divided into three parts. I call these parts *levels GO, GRO, and GR*. The part Games & Openness (GO) overlaps with level OA, the part Games & Rules (GR) overlaps with level RR, and GRO is their conjoint area. Thus GRO stands for a situation where the teacher simultaneously supports the methods of discussions and games, rules and routines, and open-approach.

Figure 1 illustrates the situation. As the three parts GO, GRO, and GR partly overlap each other, the *level DG* can be as an area of development of the beliefs and conceptions about mathematics teaching. What is essential is the *different objective for emphasis* that the students teachers give to methods of instruction.

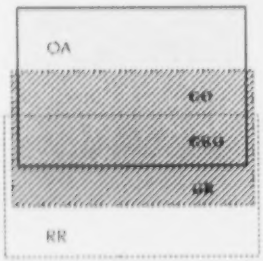

Figure 1: Three (five) levels of the development of beliefs about teaching mathematics.

These differences of emphasis could be detected in the interviews with the selected 12 student teachers and in following their practica. Using the

values of the levels OA, DG, and RR, and the interviews, 11 of the 12 student teachers could be clearly grouped into one of the five categories above.

Math memories of prospective teachers. I listed the 12 interviewed student teachers according to their assessed openness, and then pseudonyms were employed for them. In the following I give as examples statements from the transcribed interviews. Special attention was paid to the emotional memories of the students' math teachers. One exciting finding for this study was hints revealed for correlation between harshness, severity and sharpness of the math teachers they remembered, and with the student teacher's distrust of the Open-Approach method and agreement for Rules and Routines method. For each students the z-values for the levels are given in order OA, DG, and RR.

Anna (-1.2, -.22, -.21), RR
I have a small horror of math. ... There is nothing creative in math. ... A book is given to you. Here are the problems. Calculate number one, two, three, four, and five. You don't have any other choice than to do the problems, and there is only one right answer to the problems.

Beth (-.96, -.22, -.21), RR
I had a very strict teacher and I was very afraid of him. He did not let anybody ask him anything. ...in the evening I cried, and cried and my dad tried to teach and teach, but it did not help. ... In the secondary school I had a teacher who just worked at the blackboard with his back towards the student and spoke in mumbles.

Eva (-.68, .63, .15), GR
In the primary school we only calculated, and calculated the problems from the book. Then for a period I felt math was really difficult. In the lower secondary school I liked math more.

Ian (.95, .20, 2.28), GRO
My primary school teacher was the headmaster, and a very busy man, he gave us the problems, and then came to check if they were done. We proceeded precisely according to the text of the book. My lower secondary school teacher was more demanding. I think his discipline was too strict, and thus embittered the pupils' attitudes towards math.

John (1.22, .20, -.21), GO
Primary school math ... it was filling the math book, according to my memories not much else. Quite schematic. In the secondary school math was a greater challenge. But I don't remember that even there the teacher would have illustrated anything.

Karin (1.22, -.22, -.91), OA
In primary school we had a really nice teacher. She told us wonderful stories. In first grade everybody had buttons on safetypins for addition and subtraction problems. ... In lower secondary school we had a really sharp math teacher. Many were afraid of her, but I liked her. She was very accurate, and taught very well. Then I liked math

Laura (.13, -1.5, -.21), OA

From the primary school I have only good memories. I had several teachers - an old lady, a prospective young female teacher, a preoccupied man - the best memory is that I once was allowed to teach the rest of the class. In the upper secondary school I had a very good teacher. His personality was great, and he used all kinds of experiments.

Conclusion

What is the strength of these educational memories in the long run? Even during this short time in teacher education the interviews, essays and the two Likert-scale inventories duplicate changes towards a more positive and open approach towards teaching mathematics. How these views are realised in the future instructional practices will be a most interesting matter.

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Causes of being bad in Mathematics as seen by Pupils of the German Gymnasium

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Introduction

In this article I refer to the qualitative and quantitative research I made in 1995 and 1996. First I describe some conceptions of a girl who had had private lessons a few years before I made an interview with her to show the dimensions in which the causes are seen by herself. Afterwards further experiences of this pupil with learning mathematics are added to point out the context in which bad marks had arisen and in which way this pupil avoids further problems today. One will realize that occasions and causes are separated from other problematical experiences this pupil had in lessons, and these are not seen in a causal connection. Therefore it seems justified to record a map of factors which influenced bad performance. The causes this pupil has seen will be part of this map.

This case-study will afterwards be compared with results taken from an opinion poll. One can see in which categories pupils of the German Gymnasium think about the causes of being bad in mathematics in general and what importance they attribute to these categories.

Extracts from an interview with Daniela

Daniela is a pupil of grade 11 and got private lessons for half a year (grade 8 or 9).

a) Motives for ordering private lessons put forward by Daniela:

I had a lot of bad marks (5).
I realized, that I was unable to remember units of grade 5 and 6.
I had gaps particularly when doing fractions with big terms.

b) Causes of becoming bad seen by Daniela:

I had not been attentive at school.
I had not done my homework in lower grades, particularly in grade 7.
I had other interests instead of catching up on lessons.
I was unable to catch up on lessons later.
I had my first boy-friend.
I met friends instead of doing homeworks.

c) Experiences and assessments

former experiences (past and perfect tense): Teachers wrote on the black board the whole time. Pupils had to write, too. So one was unable to be attentive and was unable to work actively.	added assessments: I was not able to keep up with lessons because I could not concentrate. I did not feel like doing repetitions. I did not complete the exercises if I did not find solutions at once. I had problems with algebra, particularly with binomial formulas.
actual and global experiences (present tense): Many pupils are not able to learn by heart, some are not able to grasp things quickly.	added self-assessments and strategy: I still have difficulties in learning by heart because of lack of concentration. So I make some cribs. I have no difficulties in understanding.
further self-assessments and strategies: I may be motivated by good marks. I am not motivated yet. I work actively in lessons only to balance the bad marks I get in written tests. I am attentive if there is some fun. I do not say anything if there is no fun. (addition: It's fun if I am able to work actively or I know something other pupils do not know.) I do my homework and I am active in lessons now. I am quite lazy and have not prepared the last grade test. (conclusion: I have good marks only because of my attention and my homework.)	

All quotations are transformed and then translated by the author.

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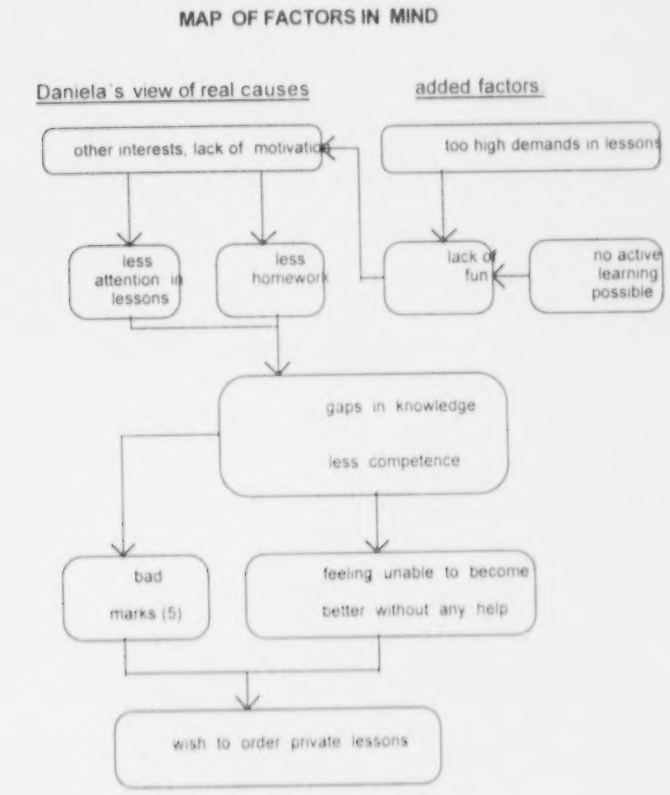
MAP OF FACTORS IN MIND
Daniela's view of real causes
added factors
other interests, lack of motivation
too high demands in lessons
less attention in lessons
less homework
lack of fun
no active learning possible
gaps in knowledge
less competence
bad marks (5)
feeling unable to become better without any help
wish to order private lessons

The arrows show influencing factors of becoming bad

Daniela is quite self-critical. She attributes her bad marks to her own behavior and lack of interest. The teachers' methods in lessons are only in passing described as a problematic factor. But Daniela is successful now because she assumed a new strategy.

Extracts from an opinion poll analysis

The opinion poll took place in the beginning of 1995. There were 111 pupils from grade 11 to 13 of a gymnasium I asked. I used questionnaires with only one question referring to this topic: What do you think are the causes why private lessons in mathematics are so often ordered by

pupils and their parents? - The formulation of this question was quite broard, because I did not want to lead them in any way. The answers could be formulated freely, in short sentences or in the form of keywords. 107 questionnaires could be analysed.

Number of all participants: N=107 pupils

	p. in grade 11	p. in grade 12	p. in grade 13	sums
p. of basic course	17	22	18	57
p. of higher course	11	22	17	50
sums	28	44	35	107

Experiences with private tuition:

Number of pupils who got private tuition for some time:	NHS=22
Number of pupils who have taught private lessons for some time:	NHL=34
Number of pupils who have done both: all from basic courses	NHS/L=3,
Number of pupils without experiences in private tuition:	54

interesting combinations:

NHS in basic courses: 18	(31,5 % of all p. in basic courses)
NHS in higher courses: 4	(8 % of all p. in higher courses)
NHL in basic courses: 13	(22,8 % of all p. in basic courses)
NHL in higher courses: 21	(42 % of all p. in higher courses)
particularly: NHL in the higher course of grade 13: 11	(65 % of this course)

The analysis of all statements requires a system of categories. The categories are taken from a very simple model of communication in school lessons. They are called

1. kinds of objects of mathematical lessons
2. teacher´s part as the organizer of learning
3. pupil´s part as a learner
4. other aspects.

Results:
Aspects of the 1. category are mentioned by 50 of 107 pupils (47%).
Aspects of the 2. category are mentioned by 56 of 107 pupils (52%).
Aspects of the 3. category are mentioned by 76 of 107 pupils (71%).
Aspects of the 4. category are mentioned by 26 of 107 pupils (24%).

Most of all pupils see problems in pupil´s personality and behavior. Half of them see problems on the teacher´s side and nearly half of them

see problems in the objects of mathematical lessons. A quarter of all pupils see causes outside the process of learning.

Secondly these 4 categories are divided to sub-categories

1. kinds of objects in mathematical lessons:
   a) topics, distance from reality, ...
   b) problem solving, ways of thinking in mathematics, ...
   c) principle of building up more and more knowledge, complexity, ...
2. teacher's personality and behavior as the organizer of learning:
   a) inadequate methods
   b) inadequate motivation, generall incompetence, lack of educational ability
   c) organizational deficits: number of pupils in class, not enough time to learn
3. pupil's personality and behavior as a learner
   a) motivational aspects: lacking diligence, insufficient interest, resignation, ...
   b) lack of talents, too high demands, ...
   c) problems in revising lessons of lower of grades
4. other aspects
   a) lack of insight in the importance of mathematics at school
   b) general attitudes to the importance of mathematics
   c) social problems: teacher-pupils, pupils-pupils, pupils-parents

Distribution of answers (A) in 4 categories: A = 290

1	2	3	4
67 (23%)	87 (30%)	105 (36%)	31 (11%)

The categories are seen in the same order of precedence. Causes are more often seen in pupils' and teachers' personality and behavior than in other aspects, particularly causes are not so often seen in the objects of mathematical lessons.

Distribution of answers (A) in sub-categories: A = 290

1a	1b	1c	2a	2b	2c	3a	3b	3c	4a	4b	4c
19	25	23	31	31	25	43	41	21	9	16	6
6,5%	8,6%	7,9%	10,6%	10,6%	8,6%	14,8%	14,1%	7,2%	3,1%	5,5%	2,1%

The sub-categories of pupils' emotional state in mathematical lessons (3a: lack of interest, resignation, ...) and a discrepancy between pupils' talents and teachers' demands (3b) are mostly seen as causes of deficits in mathematics. I think this bases on the subject-matters (1a, 1b), but these are not so often mentioned. There might be several causes why teachers are seen as incompetent to organize good lessons (2a, 2b).

In sub-category 2c the problem of less time is seen as the uppermost important organizational aspects. At last the following combination deserves attention: The consistency of mathematical knowledge (1c)

connected with the inability of revising former subject-matters independently (3c), 23+21 answers arise until 15,2 % of all answers.

It makes sense to think about other connections.

Comparison and summary

Now it is possible to compare Daniela´s self-assessments with these results. Typically she sees that the problems she had were made by herself: difficulties in elementary algebra, lack of interest connected with laziness and gaps she was not able to revise later (cat. 1, cat. 3). In her last statements she mentioned something about her motivation. If there is fun, she said, she is active. But in the phase before she got private lessons, so she told, she was unable to pay attention and unable to work, and in this case there was no fun. Here I think she describes methodological mistakes in lessons (cat. 2). But in this case she did not regard causal connections as important as pupils do in general. Her own change of attitude has been sufficient to become better.

Both studies show that pupils recognize the situation of deficits in mathematics from a centralized point of view and with their own personality and behavior in the centre of their experiences. Other factors are added. Mathematics is not regarded as being too much difficult in general. There are always other, mostly deeper factors mentioned in causal connections by the pupils at the German Gymnasium.

What are then the real causes of being bad in mathematics? - I think it is impossible to discover them by making interviews. It is just as much impossible to discover isolated causes inside pupils´ thinking, although I was able to describe one special exception of a case-study in one of the last talks. In my research I furtheron will only be able to show how pupils reflect their own situation, what their impressions are and how they react in phases of deficits in mathematics. The knowledge of their views and attitudes to learning mathematics at school are important parts of pupils´ mathematical beliefs just as well, and it is necessary to show how these are created, particularly in the context of being bad.

Some Findings in the International Comparison of Pupils' Mathematical Views

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Within a constructivist framework as a base for teaching and learning mathematics, it is found that a knowledge of teachers' and pupils' mathematical beliefs is vital if their mathematical behavior is to be understood. We will use the concepts "belief", "conception" and "mathematical view" in the sense explained e.g. in the earlier MAVI proceedings, see Pehkonen & Törner 1995.

The purpose of the research project "International comparison of pupils' mathematical beliefs" (Pehkonen 1995) is to clarify pupils' views of mathematics. But the focus lies in the comparison of pupils' mathematical views: *Are there essential differences and/or similarities in pupils' views of mathematics in different countries?* And in this pilot study, we try to provide answers to the research problem with the aid of the questionnaire data.

The realization of the pilot study

In the pilot study of the research project "International comparison of pupils' mathematical beliefs", data was gathered with the help of a questionnaire. The questionnaire used was developed for another research project, "Open Tasks in Mathematics". The purpose of the questionnaire was to clarify pupils' views of mathematics teaching. In the questionnaire, there are 32 structured statements about mathematics teaching for which pupils were asked to rate their views on a 5-step scale (1= fully agree, ..., 5= fully disagree). The questionnaire can be found e.g. in Pehkonen (1992).

Countries in question. This first part of the pilot study consisted of collecting data from about 200 seventh-graders in each country. The questionnaire has been administered in the following eight countries (the name of the local coordinator and the number of pupils' questionnaire answers in each country are given in brackets): *Estonia* (Dr. Lea Lepmann, University of Tartu; N = 257), *Finland* (Dr. Erkki Pehkonen, University of Helsinki; N = 260), *Germany* (Nordrhein-Westphalen; Prof. Günter Graumann, University of Bielefeld; N = 258), *Hungary* (Dr. Klara Tompa, Institute of Public Education, Budapest; N = 196), *Italy* (Prof. Fulvia Furinghetti, University of Genova; N = 246), *Russia* (Prof. Ildar Safuanov, University of Tatarstan; N = 206), *Sweden* (Arne Engström,

University of Lund; N = 196), and the *USA* (Georgia; Prof. Tom Cooney, University of Georgia; N = 203).

Administration of the questionnaire. This stage of the pilot study was running in 1989–94. The original questionnaire was translated from German into English by the author, and the translation was check by the USA coordinator. The versions in other languages are translated and checked by the local coordinators.

Each national representative has organized the data gathering in their own country. Usually, there are about 100 pupils from the capital city and the same amount from a smaller town in the neighbourhood of the capital. The questionnaire were filled in during a mathematics lesson, and conducted by the mathematics teacher.

In large countries, the data collection has happened only in one state, e.g. in Georgia /USA, in order to be comparable with smaller countries. And in the countries with parallel school system, as Germany, the data has been gathered from all school forms.

Comparing results from the questionnaire

Here, we are looking for similarities and differences between countries in question conserning all items. Consensus levels[3] (Table 1) give a good measure for agreement within a country. Since there are so many statistically significant differences between countries, we will focus on simililarities, i.e. items with no significant differences (Table 2).

In the following, we will use the following abbreviations: EST = Estonia, FIN = Finland, GER = Germany, HUN = Hungary, ITA = Italy, RUS = Russia, SWE = Sweden, and USA = the United States.

Consensus levels of responses. Here, we consider agreement percentages of the responses in each country separately, and check, whether they have rearched any of the consensus levels. In Table 1, each item is given with its consensus level.

[3] The responses in some items showed a very high degree of consensus. For further analysis of the responses, the original response scale (1—2—3—4—5) was reduced by combining the two response values at the extreme ends of the scale, and thus yielding a three-step scale of: agree (1 or 2) — neutral (3) — disagree (4 or 5). In the analysis and interpretation of the responses, the terminology for the consensus level was used as follows: We say that the responses to a statement are in *complete consensus*, if at least 95% of the test subjects' views were on the same extreme end of the scale; *consensus*, if at least 85% but less than 95% of the test subjects' views were on the same extreme end of the scale; *almost consensus*, if at least 75% but less than 85% of the test subjects' views were on the same extreme end of the scale; *lack of consensus*, if less than 75% of the test subjects' views were on the either extreme end of the scale.

Table 1.The level of consensus on pupils' responses to the questionnaire statements (cc = complete consensus, c = consensus, ac = almost consensus, and Σ = the number of items in consensus).

Items	FIN	HUN	SWE	EST	USA	GER	ITA	RUS	Σ
1: mental calculations	c	c	c	c	-	ac	ac	ac	7
2: right answer ... more important than the way	-	-	-	-	-	-	-	-	-
3: mechanical calculations	ac	-	-	-	-	c	-	-	2
4: pupil ... guess and ponder	ac	-	-	ac	-	-	-	ac	3
5: everything ... expressed ... exactly	-	-	-	-	-	-	-	c	1
6: drawing figures	-	c	-	-	-	-	ac	ac	3
7: right answer ... quickly	-	-	-	-	-	-	-	-	-
8: strict discipline	c	-	-	c	-	-	-	-	2
9: word problems	ac	ac	-	-	-	ac	c	ac	5
10: there is ... procedure ... to exactly follow	-	ac	-	ac	-	-	-	-	2
11: all pupils understand	ac	cc	cc	c	-	c	-	ac	6
12: learned by heart	-	-	-	-	-	-	-	-	-
13: pupils ... put forward their own questions	ac	c	-	ac	ac	-	c	-	5
14: pocket calculators	-	-	-	cc	ac	-	-	-	2
15: teacher helps ... when ... difficulties	ac	-	ac	c	ac	ac	-	ac	6
16: everything ... reasoned exactly	-	ac	-	c	-	-	ac	c	4
17: different topics... taught separately	-	-	-	-	-	-	-	-	-
18: repetition as much as possible	-	-	-	-	-	-	-	ac	1
19: tasks ... have practical benefit	c	ac	ac	cc	ac	-	ac	c	7
20: only ... talented pupils can solve (disagreement percentages)	(ac)	-	-	-	(ac)	(ac)	-	-	3
21: it could not always be fun	-	-	-	-	-	-	-	-	-
22: calculations of areas and volumes	ac	ac	ac	-	-	-	ac	-	4
23: it demands much effort	-	-	-	-	-	-	-	-	-
24: there is ... more than one way	c	ac	ac	ac	c	ac	ac	-	7
25: learning games	-	-	-	ac	c	-	-	ac	3
26: teacher explains every stage exactly	-	ac	ac	ac	-	c	-	-	4
27: pupils solve tasks ... independently	-	-	-	-	-	ac	-	-	1
28: constructing of ... concret objects	-	-	-	-	-	-	ac	-	1
29: as much practice as possible	-	ac	-	-	-	ac	-	ac	3
30: all ... will be understood	ac	c	-	c	-	c	-	ac	5
31: pupils are working in small groups	c	ac	-	ac	ac	ac	ac	-	6
32: teacher ... tells ... exactly what ... to do	-	-	-	c	-	-	-	-	1

In three items (1, 19, 24), consensus levels were rearched by seven countries, and in three further items (11,15, 31), six countries resulted consensus. Furthermore, there was a lack of consensus in each country in six items (2, 7, 12, 17, 21, 23).

Similarities in pupils' views. When checking the differences between the country means with the Mann-Whitney U test, we found that there were more items with statistically significant differences than those without of such a difference. Hence, we decided to concentrate on similarities. Table 2 shows the amount of similarities between the countries, i.e. the amount of items where the Mann-Whitney U test was not showing a statistically significant difference (on the 95 % level).

Table 2. The number of similar items between the countries.

	FIN	HUN	SWE	EST	USA	GER	ITA	RUS
FIN								
HUN	9							
SWE	15	10						
EST	9	7	10					
USA	8	7	7	7				
GER	12	10	10	7	7			
ITA	9	14	14	4	7	5		
RUS	6	8	8	7	7	8	10	

The number of the similarities, i.e. items without a statistically significant difference, varies between 4...15. The biggest number of similarities (15) is between Finland and Sweden which is not surprising, since these two countries have long time been developing their systems according to similar ideals. Instead of that, the big numbers of similarities (14) between Italy and Hungary as well as between Italy and Sweden are cumbersome. If we consider only the biggest numbers of similarities (≥ 10), we may construct Chart 3.

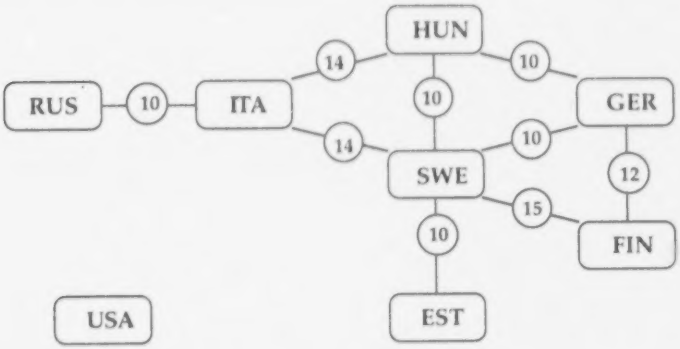

Chart 3. The numbers of the biggest similarities (≥ 10) between the countries.

The chart shows that the European countries form a cluster, whereas the US is situated totally alone. In addition, we see that Sweden has the most of the similarities with other countries. Will this show that Sweden has taken (and amalgated) many ideas from other countries? Finland had a lot of similarities with Sweden, but also with Germany, and these could be explained with common history and culture. Instead of that, a surprising point is that Russia, Hungary and Estonia do not have many similarities, althrough they had a long period of common politics, also in education. It is worthwhile noting that if we change in Chart 3 the limit of acceptance (number of similarities ≥ 10), the chart will change drastically.

Some questions which arises automathically are: In which items of the questionnaire are the similarities? Are there some items in which there are more similarities than in others? When answering these questions, we might try to sketch a common view of mathematics teaching for pupils from these eight countries: During mathematics lessons, there should be also small group working, and the teacher should help when there are difficulties. In doing mathematics, the right answer is not more important than the way of solving. Tasks in school mathematics are not only for talented pupils, and doing mathematics could not always be fun.

Endnote

The number of the differences between the countries is big. Only in 4–15 items of the 32, are the differences not statistically significant (on the 95 % level). When checking, in comparison, the differences between boys and girls in these countries, there are, as a rule, only about the same number of items with a statistically-significant difference. Thus, the differences between countries are much bigger than within a country (e.g. between boys and girls).

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Some observations concerning pupils' views
on mathematics teaching in Finland and Tatarstan (Russia)

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Introduction

Within a constructivist framework as a base for teaching and learning mathematics, it is found that a knowledge of teachers' and pupils' mathematical beliefs is vital if their mathematical behavior is to be understood. Belief systems shape cognition, even though some people may not be consciously aware of their beliefs. Conceptions are higher order beliefs. One variation of conceptions are views. Views are very near to conceptions, but they are more spontaneous, and an affective component is more emphasized in them.

In this paper, the comparative survey of seventh-graders' views concerning mathematics and mathematics teaching in Finland and Tatarstan(Russia) will be presented. This paper forms a part of the large project (Pehkonen 1995) where the main question is: Are there essential differences in pupils' views of mathematics teaching in different countries? The following research problems are derived from the aim of the survey:

(1) What are pupils' views of mathematics teaching in each country?
(2) What are the differences and similarities in these views between Finland and Tatarstan (Russia)?

The necessary data were gathered with the help of a questionnaire developed for an earlier project (Pehkonen & Zimmermann 1990). In the questionnaire, there are 32 structured questions about mathematics teaching, i.e. statements for which the pupils were asked to rate their views on a 5-step scale (1= fully agree, ... , 5= fully disagree). The questionnaire is published e.g. in Pehkonen (1992).

The questionnaire was translated into Finnish and Tatar languages by the authors. The Finnish sample was compounded of 15 grade 7 classes from Helsinki and Järvenpää (a small town about 40 km to the north of Helsinki), altogether 255 pupils. The teachers gathered the information in the middle of autumn 1989, letting pupils fill in the questionnaire at the end of their mathematics lesson.

The Tatarian sample was compounded of 8 grade 7 classes from two schools. The one school was the Tatar Gymnasium No. 2 with pupils mostly of rural origin, and the other school was the Russian school No. 44 with pupils mostly of urban origin. Altogether, there were 206 pupils. No one of the schools is a "mathematical" one. The teachers gathered in-

formation in the spring 1995. They let the pupils fill in the questionnaire at the end of their mathematical lessons.

When considering the results of the questionnaire, the statistics used were mainly percentage tables. Since the data collected is on the level of ordinal scale, we have used nonparametric statistics to check a possible statistical significance of differences between countries. The Mann-Whitney U test used is an equivalent to the ordinary (parametric) t-test on an interval scale. The StatView-program on the MacIntosh computer was used for the analysis. In the text, usual abbreviations for the significance levels are used: three stars (***) mean that the error percentage p is smaller than 0.1 %, two stars (**) that 0.1 % ≤ p < 1 %, and one star (*) that 1 % ≤ p < 5 %.

For further analysis of the responses, the original response scale (1—2—3—4—5) was reduced by combining the two response values at the extreme ends of the scale, yielding a three-step scale of:

agree (1 or 2) — neutral (3) — disagree (4 or 5).

In the published paper (Pehkonen 1993), the concept "consensus level" was introduced. In the analysis and interpretation of the responses, the terminology for the consensus level was used as follows:

- *consensus*, if at least 85% of the test subjects' views were on the same extreme end of the scale;
- *near consensus*, if at least 75% but less than 85% of the test subjects' views were on the same extreme end of the scale;
- *non-consensus*, if less than 75% of the test subjects' views were on the same extreme end of the scale.

In some cases we consider also consensus in disagreement (i.e. the number of responses 4 and 5).

Analysis of consensus levels

In the Table 0, we present the consensus percentages, i.e. the percentages of responses showing agreement (i.e. 1 = fully agree or 2 = agree) in Finland and Tatarstan (Russia) separately and then together. With the aid of the consensus percentages, we get the rank (i.e. the position in the rank order) of each item according to the consensus percentage.

Table 0. The agreement percentages of the statements. (If the disagreement percentage is bigger, it is given in brackets).

		FIN	TAT
1	doing calculations mentally	89	84
2	right answer...more important than the way	16(64)	2(71)
3	mechanical calculations	77	48
4	pupils can make guesses,use trial and error	78	78
5	everything should be expressed exactly	30(48)	88
6	drawing figures(e.g.triangles)	67	81

7	to get the right answer very quickly	11(68)	36
8	strict discipline	91	63
9	doing word problems	76	80
10	there is... procedure to exactly follow	35(53)	53
11	all pupils understand	80	80
12	learn by memorizing rules	29(50)	50
13	pupils...put forward their own question	76	71
14	use of calculators	72	35
15	teacher help...when...difficulties	76	79
16	everything should be reasoned exactly	53	89
17	different topics... taught...separately	36	22(40)
18	as much repetition as possible	61	79
19	learning math has practical benefits	94	89
20	only...talented pupils can solve	5(83)	2(36)
21	learning math is not always fun	60	42
22	calculations of areas and volumes	76	70
23	learning math requires a lot of effort	35	66
24	there is...more than one way	89	53
25	games can be used	66	84
26	teacher explains every stage exactly	72	54
27	pupils are led to solve tasks independently	73	71
28	construct of...cocrete objects	37	56
29	as much practice as possible	64	81
30	all or as much as possible is understood	75	84
31	working in small groups	85	69
32	teacher...tells...exactly what...to do	53	44

First, we will analyze cases of the double consensus (Table 1), i.e. consensus in both countries and cases of the separate consensus (consensus only in one country). We might suppose that double consensus indicates some kind of "universality" of the correspondent view.

If the consensus percentage for a statement is high in one country, but not in the other one, we might consider such a statement to be in a sense "characteristic" for the country. Thus, we will distinguish between candidates for a "universal" and for a "characteristic" view.

Table 1. The statements with double consensus.

		FIN	TAT
19	learning math has practical benefits	94	89
1	doing calculations mentally	89	84
11	all pupils understand	80	80
30	all or as much as possible is understood	75	84
4	pupil can make guesses, use trial and error	78	78
9	doing word problems	76	80
15	teacher helps...when...difficulties	76	79

Only in one of the cases the consensus is shown in both countries: in item 19 (studying math has practical benefits: 94% in Finland and 89% in

Tatarstan). For item 1 (doing calculations mentally) we see consensus in Finland and almost consensus in Tatarstan (resp. 89% and 84%). For five more items we see almost consensus both in Finland and in Tatarstan. Probably, these seven items indicate "beliefs" which are "universal" in some sense.

Table 2. The statements with a separate consensus in Finland

		FIN	TAT
8	strict discipline	90	63
24	there is...more than one way	89	53
31	working in small groups	85	69
20	only...talented pupils can solve	5(83)	21(36)
3	doing calculations with paper and pencil	77	48
22	calculations of areas and volumes	76	70
13	pupils.. put forward their own questions	76	70

From this table, we see that Finnish pupils especially prefer strict discipline and solving problems by more than one way. Clearly, these views are "characteristic" for them.

Table 3. The statements with a separate consensus in Tatarstan.

		FIN	TAT
16	everything...reasoned exactly	52	89
5	everyth...expressed exactly	30	88
25	games can be used	66	84
6	drawing figures(e.g.triangles)	66	81
29	as much practice as possible	63	81
18	as much repetition as possible	62	79

Pupils in Tatarstan are rather more than Finnish ones in favor of statements (6,18 and 29) reflecting mechanistic approach to the mathematics teaching cultivated in Soviet schools. In two items reflecting exact reasoning and explanations the differences between pupils in two countries are surprizingly big and, certainly, corresponding beliefs are characteristic for pupils in Tatarstan.

Table 4. The statements with a double non-consensus: the agreement percentages (the disagreement percentages are given in brackets, if they are bigger).

		FIN	TAT
27	pupils are led to solve tasks independently	73	71
26	teacher explains every stage	72	54
14	use of calculators	72	35
21	learning math is not always fun	61	42
23	learning math requires a lot of effort	35	66

32	teacher...tells..exactly what... to do	52	44
28	constructing of...concrete objects	38	55
10	there is...procedure to exactly follow	35	53
12	learn by memorizing rules	29	50
17	different topics...taught separately	35	21
7	to get the right answer very quickly	11	36
2	right answer... more important than the way	16(64)	2(71)

The lowest level is shown in item 2 (getting the right answer is more important than the way of solving the problem. It was observed that this item is near to "universal" in disagreement.

Focus on differences between Finland and Tatarstan

Further, we are using another approach to the analysis of our data, arranging items according to the absolute values of differences in the agreement percentages between pupils in Finland and Tatarstan (Table 5).

Table 5. The differences in agreement percentages in Finland and Tatarstan (the percentages in Table 0 are subtracted in the given order).

		diff.
(A)	ITEMS WITH ABSOLUTE VALUE OF DIFFERENCES 25 % OR MORE	
5	everything should be expressed exactly	-58
14	use of calculators	+37
16	everything should be reasoned exactly	-37
24	there is...more than one way to solve a problem	+36
23	studying math requires a lot of effort	-31
3	doing calculations with paper and pencil	+29
8	strict discipline	+27
7	to get the right answer quickly	-25
(B)	ITEMS WITH ABSOLUTE VALUE OF DIFFERENCE BETWEEN 10 % AND 25 %	
12	learn by memorizing rules	-21
21	studying math is not always fun	+19
10	there is a procedure...to exactly follow	-18
25	games can be used	-18
26	teacher explains every stage exactly	+18
29	as much practice as possible	-18
18	as much repetition as possible	-17
28	constructing of... concrete objects	-17
20	only...talented pupils can solve most of problems	-16
31	working in small groups	+16
6	drawing figures (e.g. triangles)	-15
2	answer is more important than the way of solving	+14
17	different topics should be taught separately	+14

The rest of items have absolute value of difference less than 10 %. We may suppose that items in the group (A) indicate important specific traits of teaching /learning in each country.

The greatest difference is shown in items 5 and 16. Thus, the pupils in Tatarstan believe much stronger than those in Finland that everything ought to be expressed and reasoned as exactly as possible. Large differences showing more positive attitudes of pupils in Tatarstan are seen also in items 23 and 7. These figures indicate the strongness of demands on pupils in Tatarstan. Pupils in Tatarstan have to solve a lot of problems at each lesson, so they have to solve them very quickly, having no time for seeking alternative solutions. The greatest differences which show more positive attitudes of pupils in Finland than in Tatarstan are shown in items 14, 24, 3 and 8. Taking into account item 2 , one can make a conclusion that mathematics teaching in Finland is in greater extent calculation-oriented. Secondly, pupils in Finland very much favor strict discipline.

Note that the Tatarstan pupils seemed to have more difficulties in deciding their viewpoint than their Finnish mates: neutral (3) is the most frequent response in nine items in Tatarstan (3, 7, 10, 12, 14, 17, 20, 21, 32) and only in four items in Finland (10, 17, 23, 28). The pupils in Tatarstan are more restrained in their self-expression and seem to have rather low self-confidence. This fact might be explained by the authoritarian style of teaching in Tatarstan.

Deeper analysis related to some aspects.

In this section, we will try to analyze responses to separate groups of questions related to some interesting aspects of teaching and learning mathematics. We choose three aspects (problem-orientedness, pupils' independent work, demands on teachers and pupils) which are rather important for the effectiveness of teaching, and for the development of creativity of pupils' minds.

Since problem-orientedness is in items 2 and 26 characterized by disagreement, not by agreement, these two items are converted in Table 6, i.e. we consider consensus in disagreement calculating the number of responses 4 and 5.

Table 6. Consensus percentages, differences in consensus percentages and the significance level of differences (Mann-Whitney U) in the statements reflecting problem-orientedness.

		FIN	TAT	diff.	M-W
-2	right answer...not more important than the way	64	71	-7	**
4	pupils can make guesses, use trial and error	78	78	0	**

24	there is... more than one way	89	53	+36	***
~26	teacher doesn't explain every stage exactly	13	18	-5	**
27	pupils are led to solve tasks independently	73	71	+2	-

From this table, we may conclude that in both countries mathematics teaching is striving towards problem-orientedness, but only with a different emphasis. In Table 7, we converted results in items 15, 26 and 32.

Table 7. Consensus percentages, differences in consensus percentages and the significance level (Mann-Whitney U) in the statements reflecting independent work.

		FIN	TAT	Diff.	M-W
4	pupils can make guesses...	78	78	0	**
13	pupils... put forward their own questions	76	70	+6	-
~15	teacher doesn't help as soon as possible when difficulties	15	5	+10	-
~26	teacher doesn't explain every stage exactly	13	18	-5	**
27	pupils are led led to solve tasks independently	73	71	+2	-
~32	teacher doesn't...tell... exactly what...to do	29	21	+8	-

We see that the orientation to the independent work of pupils both in Finland and in Tatarstan are similar. But independent work is not very strongly favored by pupils, since only 3 of altogether 12 agreement percentages exceeded "almost consensus" level.

Table 8. Consensus percentages, differences in consensus percentages and the significance level (Mann-Whitney U) in the statements reflecting demands on teachers and pupils.

		FIN	TAT	Diff.	M-W
DEMANDS ON TEACHERS					
5	everything should be expressed as exactly as possible	30	88	-58	***
11	all pupils understand	80	80	0	-
16	everything should be reasoned exactly	52	89	-37	***
18	as much repetition as possible	62	79	-17	***
29	as much practice as possib.	63	81	-18	***
DEMANDS ON PUPILS					
7	right answer... quickly	11	36	-25	***
23	studying math requires a lot of effort	35	66	-31	***

The results in this table show that in mathematics teaching, demands on pupils and their views of demands on teachers are in Tatarstan much stronger than in Finland.

Discussion

Generally, views on mathematics teaching and learning in Finland and Tatarstan are rather far from being similar to each other. But we did find seven for both countries common items. With the help of these items, we might draw a universal picture on mathematics teaching and learning in Finland and Tatarstan: In mathematical content area, mental calculations and word problems have a steady place. Teaching should strive for understanding, and pupils are expecting their teachers to help them in this. Nevertheless, pupils should also have possibilities to guess as well as use trial and error. And furthermore, mathematics to be learnt should have practical benefits.

The Mann-Whitney U test shows that in most items, the reactions of pupils in Finland and Tatarstan are not correlated. Investigation of differences between pupils' answers in two countries shows also views which are characteristic for each country. For pupils in Tatarstan, most characteristic are: the importance of exact reasoning and explanations, and items reflecting strongness of demands on pupils (and teachers). For pupils in Finland most characteristic are items reflecting calculations-orientedness of learning mathematics, and favor of strict discipline and teacher-orientedness. Besides, the research revealed some peculiarities in pupils' reactions in two countries. For example, pupils in Tatarstan showed undecidedness in many items.

When discussing problem-orientedness, we may conclude that in both countries mathematics teaching is striving towards problem-orientedness, but only with a different emphasis. In the case of independent work, the results indicate that the orientation in question for pupils both in Finland and Tatarstan are similar, although it is not very strongly favored. It resulted that in mathematics teaching, demands on pupils and their views of demands on teachers are in Tatarstan much stronger than in Finland. These groups of questions are of interest for a more thorough study, e.g. interviewing a group of students. Especially of interest would be to compare results in these groups of questions for a larger number of countries.

The differences between Finland and Tatarstan are big. Only in six items of all 32, the differences are not statistically significant. When checking the differences, for example, between boys and girls in both countries, there were only seven items with a statistically significant difference. Thus, the differences between countries are much bigger than

within a country (e.g. between boys and girls). This might also give an interesting idea for continuation.

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Changing pre-service teachers attitudes towards mathematics

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Background and purpose

The interest on affective variables in mathematics education became a focus of research during the 1980s (Lester, Garofalo & Croll, 1989; Pehkonen, 1994). The domain of attitudes, however, was quite a hot area as from the 1960s and several research-review articles have been published since then (Aiken, 1976; Reyes, 1984; Pajares, 1992; McLeod, 1994). Some of the key questions investigated include understanding of emotions, attitudes, and beliefs of students and teachers, detecting individual differences, and describing their relationship with cognitive variables. Of vital interest were issues of emergence and change of beliefs, since the individual's relationship and feelings about a subject is a basic determinant of his/her learning/teaching behavior.

Though there is no consensus concerning the structure and the role of affective constructs-even key concepts are rather loosely defined-it seems that research has provided convincing documentation about the following:

- The affective domain is inseparable from the cognitive domain. Affective responses depend on one's experiences, which consist of factual knowledge, interconnected and influenced by emotions generated by the specifics of the situation under which the individual passed through that experience.
- The development of belief or attitude is the result of a long experienced-based process, but once formulated it has a degree of stability and intensity. It may change only if the individual is faced with conflicting new experiences in terms of knowledge and/or emotions..
- Beliefs, attitudes and emotions form a hierarchical scale characterized by decreasing stability, decreasing involvement of cognitive elements, and increasing level of affective components. Intensity may vary irrespective of the construct; a belief or emotion may be strong or loose, depending on the provoking situation and the degree of personal involvement or interest. Attitudes occupy the middle part of the ladder; they are fairly stable and depend, more or less equally, on cognitive as well as on emotional factors.
- Students' attitudes towards mathematics were found to be quite satisfactory at the early primary level, but get less positive while

students grow older and become rather negative as they proceed to the secondary and the high school.

Teacher's attitudes towards mathematics and teaching of mathematics play a significant role in shaping his/her instructional practice and consequently in influencing pupils' attitudes, motivation, and achievement (Pajares, 1992; Thompson, 1992). Gibson and Dempo (1984) examinèd teacher beliefs, academic focus, and behaviors; they found that teachers with high self-efficacy beliefs engage in practices that are associated with high achievement gains. Fernandes (1995), summarizing the results of a host number of studies concluded that:

- Teachers and their formative experiences in mathematics emerge as key players in the process of teaching, since what a teacher does in the classroom reflects his/her belief system.
- Most teacher education mathematics programs do not take into account what beliefs and attitudes the candidates bring with them.
- Studying teachers' views, attitudes and beliefs provides information necessary to teacher-educators in the process of designing teacher education programs.

Though understanding of factors that enhance or diminish prospective teachers' attitudes and self-efficacy provides a foundation for re-examining preparatory programs, only few studies have been reported on the relationship between teacher characteristics and self-efficacy beliefs. Moreover, no convincing answers have been produced to important issues concerning teachers' attitudes towards mathematics such as "How do these attitudes evolve" and "How can they be affected". Both questions are of paramount significance, if some kind of specific intervention is to be undertaken.

A number of studies indicated that many students develop negative attitudes at the high school (Smith, 1988); some of those students choose to become primary school teachers, and eventually teach a subject they dislike. Unconsciously, these teachers influence students' attitudes negatively and the system is thus moving into a vicious circle. The crucial question is how and when could this circle be broken down. It has been noticed (Schoenfeld, 1994) that the pre-service period is appropriate, because student-teachers are then exposed to long-lasting experiences, organized under the leadership of experts in mathematics education. It is, therefore, important for teacher-trainers to evaluate their programs with respect to cognitive and affective outcomes. Understanding the impact of individual and institutional characteristics on student-teachers' beliefs, a knowledge base is provided for educators

to develop programs, which may enhance students' experiences and improve the likelihood to become successful teachers of mathematics.

Although there appears to be a desire to make better use of history in the teaching of mathematics, very few studies have investigated its usefulness in the teaching of mathematics. History of mathematics is among the courses offered by several European universities but, to the best of our knowledge, there are no reports of its effectiveness for teacher preparation and particularly in relationship to beliefs. Stander (1991) found that historical topics used as enrichment materials made no difference in the attitudes of prospective primary teachers; most subjects stated that they enjoyed reading about great mathematicians connected with mathematics they were studying, but confessed that they would be reluctant to spend time in search of the historical development of mathematical concepts or ideas.

Thus, this study focused upon the effect of the overall mathematics preparatory program for elementary education majors rather than on a particular course. Furthermore, this article adds to the body of knowledge about changes in students' attitudes, reporting the results of a longitudinal study, which took into account specific environmental variables and some well established outcomes of earlier studies. The aim of the present project was:

- To develop and implement a preparatory mathematics program for primary teachers, and test its effectiveness in improving prospective teachers' attitudes towards mathematics.

Methodology

The research design of this study was of the type "pre-test, treatment, post-test". The subjects' attitudes' were measured prior, during, and after the implementation of the program, through a period of three years.

Some characteristics of the Program. Previous research in the same culture showed that high school students and prospective teachers have rather negative attitudes towards mathematics (Philippou, 1994). Taking into account related entering characteristics, the program designed at the University of Cyprus aimed at providing students opportunities to acquire mathematical content in such a way that they would feel comfortable with fundamental concepts and methods and at the same time improve their confidence in doing mathematics. Special attention was taken to provide for experiences which would i) help the students reconsider their views about the nature, the usefulness, and the difficulty of learning mathematics, and ii) guarantee success experiences, if possible, for all participants.

The program consisted of two content courses, organized along the historical-developmental lines, taking advantage of the social and

cultural environment, and one method course. It was hypothesized that Greek students would have special incentive to read mathematics related to K-6 curriculum, mostly developed by Greek mathematicians. It was envisaged that following the evolutionary process of mathematics would contribute not only to understanding of concepts but also to developing of a sense that mathematics is not a fixed and finished product, but a rather constantly changing result of human activity. The prospective teacher would be given the opportunity to appreciate, for example, the power of the "place value" property by following the struggle of Babylonians and Greeks to derive algorithms and do useful calculations in the hexadecimal and the alphabetical system respectively. Working on a few dozens of different proofs of the Pythagorean theorem, the student was expected to overcome the long cherished myth that each mathematical problem has always one single solution. Similarly, a study of the three famous problems of antiquity will help realizing that the existence of solutions of a problem depends on the "rules of the game" and that failure is not necessarily futile. Finally, it was envisaged that following through some of the successes and failures of some great mathematicians would function as a powerful motive encouraging patience and persistence, leading students to thought provoking experiences, to intrinsic motivation and hence to improved attitudes. The courses were taught in two one-hour lectures and one and a half hours of group activities, aimed to offer students opportunities for success.

Instrumentation. Three complementary scales were used: the Dutton's Attitude Scale (Dutton, 1988), slightly adapted to suit the cultural environment, the Self-rating Scale, and the Justification Scale. Dutton's scale consists of 18 statements reflecting feelings towards mathematics ranging from extreme negative to the most positive attitudes (Dutton had assigned weighting factors from 1 to 10.5). The subjects were called upon to endorse those items in agreement with their own feelings. The Self-rating was a linear uni-dimensional scale, on which the subjects indicated their overall feelings within a range from 1 (absolute detest) to 11 (real love of mathematics), with 6 indicating neutral attitudes. The Justification scale consisted of two parts with ten statements each, providing reasons for liking and for disliking mathematics respectively. The instrument was administered to the subjects in 1992, just before the commencement of the first mathematics course (phase 1), after the completion of the first course (phase 2) and finally in 1995, when the whole program of three courses was completed (phase 3).

Finally, ten semi-structured interviews were curried out, some months after the end of the third course, to elicit additional information. The interviewees were encouraged to give their evaluations on: i) Their feelings about mathematics prior to University, and ii) the effectiveness

of the mathematical experiences they had in the University related to their attitudes.

Participants. The subjects were all prospective primary teachers enrolled at the University of Cyprus in 1992, the first year of its operation (N = 162); those who completed the first course in 1993 (N = 134), and finally those who completed all three courses in 1995 (N = 128). Their age at the beginning of the course varied from 18 to 21 years. Entrance requirements at the University of Cyprus are based on examinations and usually freshmen come from the top 25 % of high school graduates. Nonetheless, about one third of the prospective teachers come from the Classical Section, while two thirds do not take mathematics (optional subject) at the entrance examinations.

Results

The results are discussed with respect to the proportion of subjects' who endorsed each item of the three scales in each of the three phases. To detect patterns in attitude change and highlight important differences among the three phases i) the x2-test was used and ii) the Median Polishing Analysis was also applied to responses on the Dutton Scale.

The pre-test measurements (see Appendix) revealed an alarmingly large proportion of students (24 %) who "detest mathematics", while comparative percentages endorsed negative statements, such as "had never liked mathematics" (28 %), and "do not feel sure of myself in mathematics" (47 %). The same pattern of responses appeared also in the self-rating scale where 33.5 % of the subjects disclosed that they had negative feelings. From the Justification scale, the subjects seemed to like mathematics mostly because "it develops mental abilities" (47 %) and "it is practical and useful" (39 %); the next two were "is necessary for modern life" and "interesting and challenging". Most often mentioned reasons for disliking mathematics "was afraid of them" (29 %) and "poor teaching" (27 %), followed by "lack of teacher enthusiasm" and "lack of understanding".

Concerning attitude change, the comparison between responses in the three phases revealed significant differences on 14 out of 18 statements of the Dutton's Scale and on nine out of ten items of the Liking part of the Justification Scale. Significantly fewer subjects endorsed negative statements and more subjects endorsed positive statements at phase-3 than in phase-1. For instance, the proportion of those who "detest mathematics" dropped from 24 % to 12 % and of those who "never liked mathematics" from 28 % to 18 %. Conversely the proportion of those who "would like to spend more time at school working on mathematics" raised from 15 % to 18 % and those who "enjoy working and thinking about mathematics outside school" from 20 % to 40 %. The positive

change in attitudes was also affirmed by responses on the Self-rating Scale. The proportion of subjects with negative overall feelings towards mathematics dropped from 33.5 % at the entering stage to 21 % in phase-2 and phase-3, a really remarkable change. On the other side, the proportion of subjects with favorable attitudes from 59 % at the pre-test raised to 71 % at the post-test. Significant differences were also found on the Liking part of the Justification scale e.g. in statements like: "necessary for modern life" (35 % to 76 %), "it develops mental abilities" (47 % to 72 %)", its logical" (29 % to 50 %). On the reasons for Disliking mathematics scale differences were found only on two items: More students were convinced about their teachers' "lack of enthusiasm", and fewer that "mathematics are not related to everyday life".

The Median Polishing Analysis partitions two-way tables into four interpretable parts: the grand effect, the row effects, the column effects, and the interactions of rows by columns. The grand or overall effect indicates the typical response across all the items; the row effect tests for differences between responses in different phases; the column effect reveals relative differences among the level of endorsement of items, and the cells (interaction between rows and columns) contain the Residual effects. The latter represent the extent to which endorsement of these items cannot be explained by differences among phase. or items, but represent unique patterns of responses by subjects to particular items. For the application of this analysis, the Dutton's scale was partitioned into three major parts. The first focused on the *satisfaction* from mathematics, i.e., the extent to which subjects view themselves as interested, motivated, and able to do mathematics; the second focused on the mathematics *anxiety*, i.e., the extent to which they feel insecure and fear in doing mathematics, and the third one on the *usefulness* of mathematics in daily life.

The grand effect from the median polishing analysis of data of the eight items in the satisfaction part was 18 % endorsement (see Table 1), meaning that, on the whole, students did not tend to endorse the sentiments expressed in the items. Changes of attitudes, however, during the three phases, were rather large. When entering the University (phase 1), the subjects were least likely to endorse the items on the satisfaction scale, (the effects were found -4.5), after the first mathematics course, they became 0, and after exposure to the whole program (phase 3), they were found to be 22, a rather dramatic shift from earlier phases, indicating that the subjects expressed significantly better feelings of satisfaction than in the first two phases.

According to the method, column effects within the range ± 5, are negligible and only effects whose absolute values were greater than 10 are discussed. In this case, there are large differences in the degree to

which students agree with the opinions stated in the satisfaction scale. Students affirmed the notion that they enjoy mathematical problems when they can solve them (item 11, effect size 18) and that they sometimes enjoy the challenge of mathematics (item 12, with effect size 12.5). It is interesting to note that students were less positive about spending more time in school working mathematics (item 15, effect size = -7).

Table 1. Median Polishing Analysis for the Satisfaction Scale.

Items	9	11	12	14	15	16	17	18	Row Effects
Phase 1	2.5	0	-0.5	0	0	-2.5	- 4.5	0	-4.5
Phase 2	0	-4.5	0	-3.5	2.5	0	0	0.5	0
Phase 3	0	10.5	10	6.5	-13.5	0	-10	-19.5	22
Column effects	1.5	18	12.5	6	-7	1.5	-2.5	-3	Grand effect = 18

The mathematics anxiety part consisted of five items intended to measure the extent to which students find working with mathematics an unsettling or frightening undertaking. In this part of the scale (Table 2) endorsement means a higher level of anxiety or fear. The overall effects for these items was 12 % which represent a poor endorsement of the ideas portrayed by these items. The row effects, i.e., the differences between the phases were very small indicating that students responded in the same way during the three phases of the study. The column effects were also very small except in the case of item 6 (effect size = 14) indicating that students in phase 3 became more confident in doing mathematics.

Table 2. Median Polishing Analysis for the Mathematics Anxiety Scale.

Items	1	2	3	4	6	Row Effects
Phase 1	4	3	-7	0	0	0
Phase 2	0	0	0	2	-8	-2
Phase 3	-2	0	1	-2	10	6
Column effects	-4	0	-4	-4	14	Grand effect = 12

The Utility scale with four items addressed students' perceptions of the significance and usefulness of mathematics in everyday life. The grand effect on this set of items was 25 percent (Table 3) indicating that students in general are more likely to endorse the usefulness of learning mathematics. The between phases effects were very large. In phase 1, students did not seem to endorse the usefulness of mathematics (effect

size = -2.5), while in phase 3 they responded positively to all of these items (effect size = 37).

Table 3. Median Polishing Analysis for the Mathematics and Utility Scale.

Items	5	7	10	13	Row Effects
Phase 1	1.5	12.5	-5.5	0	-2.5
Phase 2	-3	-2	0	2.5	0
Phase 3	0	0	1	-13.5	37
Column effects	-0.5	0.5	3.5	-1	Grand effect = 25

From the interviews it was evident that most of the subjects developed negative attitudes out of experiences at the high school., while the program courses helped them develop positive attitudes; they mentioned specially the contribution of the historical development of mathematics This is highlighted by the following extracts:

- "My attitudes were extremely negative thanks to my teachers. Mathematics was for me a piece of work based on getting the right answer and most of the times I could not succeed".
- "The proper way to learn mathematics was by memorizing facts and procedures ",... "any statement or answer in mathematics was either right or wrong".
- "When I entered the University I felt great relief; I was happy, thinking that I had finished with mathematics. The moment I learned that the program of studies required 3 more courses in mathematics I felt frustration. I felt that mathematics will hunt me for ever".
- "History of mathematics provided me with a variety of interesting, new, experiences.... Through the journey I realized that mathematics has always been and continues to be a useful subject.... I appreciated the efforts of people to use mathematics to solve daily problems. The course showed me that mathematics is, at least sometimes, a human activity. I felt more confident when I realized that even great mathematicians did mistakes as I frequently do".

Conclusions

The results of this study seem to confirm earlier findings about the input to the teaching profession and generate hopes on the possibility of changing attitudes towards mathematics. It was found that prospective teachers have negative attitudes towards a subject they are supposed to teach. For social reasons teaching does not attract candidates highly motivated to learn and teach mathematics and this trend is not likely to change in the future. Most of the newcomers will continue to hold unfavorable attitudes towards mathematics and it relies on the Teacher

Education Department to improve the situation. Since attitudes and beliefs are gradually shaped over the years on the basis of related experiences, the effort to improve them is a long process, which should take into account the social and cultural environment.

In the present project we took advantage of two factors which were proved to be quite decisive: the establishment of a new Department hence the possibility of designing from the beginning a mathematics program, and the historical heritage of the population under study. The developed program, based on the history of mathematics, was found to be effective in improving prospective teachers' attitudes. It produced some attitude change as evidenced by i) an increase in the percentage of responses to the satisfaction, and utility scales; ii) the improved responses of students at the Justification scale (liking part) and on the self-rating scale iii) the participants comments in the interviews. According to students' evaluations, history played a major role in this change, yet new mathematical experiences and instructor's influence cannot be ruled out. A question which remains to be answered concerns the endurance of this change and the real effect on their teaching behavior.

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Appendix

Responses on the Dutton's Scale	Phase 1 (%)	Phase 2 (%)	Phase 3 (%)
1. I detest mathematics and avoid using them all the times.	24	12	12
2. I have never liked mathematics.	28	21	18
3. I am afraid of doing word problems.	15	31	24
4. I have been always afraid of mathematics.	14	19	13
5. Mathematics is something you have to do even though is not enjoyable.	39	42	58
6. I do not feel sure of myself In mathematics.	47	35	42
7. I do not think mathematics is fun but I always want to do well in it.	62	50	60
8. I am not enthusiastic about mathematics, but I have no real dislike of it.	30	29	42
9. I like mathematics, but I like other subjects as well.	20	36	36
10. Mathematics is as important as any other subject.	32	60	64
11. I enjoy doing problems when I know how to do them.	53	69	66
12. Sometimes I enjoy the challenge presented by a mathematics problem.	43	65	61
13. I like mathematics because it is practical.	34	55	43
14. I enjoy seeing how rapidly and accurately I can work on problems	33	40	52
15. I would like to spend more time at school working on mathematics.	11	26	18
16. I enjoy working and thinking about math problems outside school..	20	40	40
17. I never get tired working with mathematics.	19	31	27
18. Mathematics thrill me and I like it better than any other subject.	15	29	16

- Note: Item no 8 was omitted from the Median Polishing Analysis as neutral.
- The items of this scale were given at a random order (16, 6, 14, 9, 13, 7, 8, 10, 5, 12, 3, 15, 1, 11, 18, 4, 17, 2).

Change in Mathematical Views of First-Year-University Students

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Ninety years ago the famous Göttinger mathematician Felix Klein started to create a series of books presenting elementary mathematics from a higher viewpoint (Klein 1908). In his preface he mentioned the later on often quoted two discontinuities[4]. Although the mathematical curricula for university students as well as for highschool students have changed much since those days, there are hints that the mentioned discontinuities are still existing, in particular on the level of mathematical beliefs.

Purpose of the study

The aim of the study was to identify and describe changes in the attitude spectra of teacher students within their first year at university in order to answer the question: Is there a real discontinuity? What are the influencing parameters of such a discontinuity. In a separate study a series of interviews were conducted with prospective teachers having left university and entering school on which the first author will report elsewhere. Here, seven students were interviewed within their first four weeks at university. Individual interviews were again arranged with the same group at the beginning of their second semester. Then we compared the 'measured' beliefs that evolved then with those verbalized in the first interview. This topic of prospective teacher freshmen has been focused by few authors (see Reichel 1991, 1992a, 1992b, Doig 1994, Malone 1996, Sander 1996 and others) during the last years.

[4] "Der junge Student sieht sich am Beginn seines Studiums vor Probleme gestellt, die ihn in keinem Punkte mehr an Dinge erinnern, mit denen er sich auf der Schule beschäftigt hat; natürlich vergißt er daher alle diese Sachen rasch und gründlich. Tritt er aber nach Absolvierung des Studiums ins Lehramt über, so soll er plötzlich eben diese herkömmliche Elementarmathematik schulmäßig unterrichten; da er diese Aufgabe kaum selbständig mit seiner Hochschulmathematik in Zusammenhang bringen kann, so wird er in den meisten Fällen recht bald die althergebrachte Unterrichtstradition aufnehmen und das Hochschulstudium bleibt ihm nur eine mehr oder minder angenehme Erinnerung, die auf seinen Unterricht keinen Einfluß hat. *Diese doppelte Kontinuität...*"

Theoretical framework

Since there is no commonly accepted standardized definition of beliefs we assume that beliefs are the compound of a person's subjective (experienced) implicit knowledge (and feelings) concerning mathematics and its teaching / learning (see Pehkonen & Törner 1996). In this sense we regard beliefs as attitudes constituting itselves through at least three components: an affective one, a behavioral one and a cognitive one. Each component (and, as a consequence, the belief as an attitude) is measured by different ways of reacting. In other words, we are in favour of the "three-components-approach" (Rosenberg & Hovland 1966) seeing attitudes as a - scarcely specified in more detail - system of cognition, affection, and behaviour (conation).

The cognitive component can be marked as subjective knowledge or, in general, as information of a person about an object. It is important to say that this knowledge does not have to be proven valid in an objective manner.

The affective component of a belief affects the emotional relationship to an object. It refers to the idea that a certain feeling or emotional state is connected with that social object an attitude is directed at.

The behavioral (conative) component of a belief which is relevant to action is readiness or tendency (probability) to act in a certain manner (a class of actions) regularly provoked by a social object. It should be mentioned that, in this case, a person is only ready to act; it is not necessary for the action to be carried out (cp. Süllwold, p. 476).

Methodology

The study is based on two series of videotaped interviews with the same seven mathematics teacher students, the first of which already has taken place in the first four weeks of winter semester 95 / 96, the second interview series was recorded in April 96. The interviews were held at the location of Gerhard-Mercator-University of Duisburg. The students were chosen by random. We asked an assistant to name seven students attending the main courses (Calculus I) of the first semester.

When working out the interviews we have being lead to a great extent by the questionnaires that Sander (1995) has introduced (see Kalesse 1996 for details), partly and independently used by Reichel.

Data Collection and Analysis

Our investigation covers two sets of interviews. The questions in the interviews covered different topics given by the categories of Ernest (Ernest 1989), namely (a) the student's conception of the nature of mathematics, (b) a student's model for teaching mathematics, (c) a student's model for learning mathematics and (d) a student's general

principle of education. In this note we focuse only some aspects of these dimensions. During both interviews the students were asked to rank fifteen items which are associated with the nature of mathematics, its teaching and learning process. The highest rank was assigned to the natural number 15, the lowest rank to number 1.

Each item was represented by a card and the students were asked to define some ranking of these cards by ordering them. The student's comments throughout this process were recorded.

		Teacher Students Level Secondary I						Teacher Students Level Secondary II							
		Ms. K 1.S.	2.S.	Ms. M 1.S.	2.S.	Mr. S 1.S.	2.S	Ms. A 1.S.	2.S	Ms. C. 1.S.	2.S	Ms. E 1.S.	2.S	Mr. L 1.S.	2.S
motivation	a/vp	11	7	15	15	15	7	14	10	15	15	15	11	15	14
comprehension	c/vp	14	14	13	9	14	15	10	11	12	1	13	5	12	15
duplicating of proofs	c/p	10	12	12	12	9	9	7	12	14	14	11	15	14	13
fun	a/vp	15	15	11	10	13	11	9	2	4	12	14	10	4	3
knowledge	c/p	13	11	7	14	11	14	12	13	9	8	6	14	9	7
good memory	b/p	4	10	8	11	7	13	13	8	13	13	12	6	7	8
own activity	c/vp	5	13	14	13	8	6	8	7	5	4	10	12	13	12
sense of achievement	a/vp	9	9	10	7	3	12	15	3	7	11	7	9	10	9
competence	c/vp	12	8	8	4	10	8	11	1	8	7	5	13	8	5
creativity	c/vp	7	4	3	8	4	1	5	6	6	5	9	7	11	4
imagination	c/vp	8	3	4	6	5	2	6	5	2	8	8	8	5	6
dull rote learning	b/n	3	1	5	2	2	5	2	15	11	9	4	1	2	2
emotions	a/p	8	8	2	3	6	3	3	4	3	3	3	3	6	11
learning by heart	b/n	1	5	9	5	1	4	1	14	10	10	2	2	1	1
fear	a/n	2	2	1	2	12	10	4	9	1	2	1	4	2	10

Further we decided to link these items with primarly the cognitive (c), the affective (a) or the behavioral (conative) (b) dimension of beliefs. Starting with the affective component, which has two outcomes of expectations (positive, negative), we differentiate the outcomes of the other variables into the degrees: very positive (vp), positive (p) and negative (n). Note that only duplicating of proofs involves a primarly mathematics-related dimension.

The results

We now take a closer, however limited view on each student in order to observe some mechanisms of their changes in attitudes towards the subjects within the first semester.

Mrs. K. Sec. I. She visited Hauptschule till grade 10. Her childhood was not pleasant which seemed to influence her career at school. She likes mathematics, but does not love mathematics. Her favorite subject would have been architecture. Not only the importance of motivation decreased, but also creativity, competence, phantasy, and knowledge. She seems to have the opinion that own activity, sense of achievement, good memory as well as dull rote learning are decisive factors.

Mrs. M. Sec. I. At school she was a very talented student in mathematics courses. Mathematics was a favorite subject of her, however dominated by the plan to become a teacher. Although she is very successful at university, she is not in favor of the kind one is learning mathematics at university. Since she was the best in her class, sense of achievement is no longer as important as it was. Instead of that she is favouring the factors: knowledge, creativitiy and seems to have become more demanding.

Mr. S. Sec. I. He visited Hauptschule till grade 10. His motivation towards mathematics depended much on the teacher. His expectation for the university studies was to get deeper insight in school mathematics. In the first interview he is afraid that he will not be capable to reach the exam. His attitude towards mathematics is neutral, on the other hand he admires mathematics. Meanwhile he has left university, however, we were lucky to get a second interview with him. He argued that the only possibility to keep track was to learn by heart which demotivated him. At school the teacher normaly would take care of each student and explains the various topics as long till all the students have acquired the theme. Thus, at university, he felt a need of being successful, however he failed. To him the variables 'creativity' and 'imagination' are no longer linked with mathematics, only the association with the variable 'fear' will remain.

Mrs. A. Secondary Level II. Realschule till grade 10. In Oberstufe she only attended a basic course. Before entering university math was her favorite subject. Her motivation towards mathematics depended much on the teacher. She states a correlation between having success and having fun in mathematics whereby success seems to be the primary factors. There is a dramatic qualitative and quantative change in her estimation of mathematics. She failed the written examinations. So there is a drastic change in her estimations of the relevant variables, e.g. learning by heart as well as fear are highly scored in the second interview. Although she describes her experience as a shock and a horror

trip with respect to affective factors, she decided to continue studying mathematics. Mrs. A. is an significant example how changes in attitudes leading in consequence to an affective self-destruction are caused by a bundle of factors. So she is pleading for a continuous consultation including psychological as well as mathematical aspects. Finally she proposed a separation of the teacher education from the Diploma study.

Mrs. C. Sec. II. Primarly she intended to become an ingenieur. She was highly motivated towards mathematics. At school she had experienced excellent math teachers. She successfully passed through both written examinations of the first semester. Although she had no major difficulties, she told that she had to work hard in the first semester. She would never advise students to study mathematics in case they have only attended basic courses at school. Nevertheless she heavily critizes the mode of learning and the quality of teaching. Only due to her unhurt and positive image of mathematics which she acquired at school she was able to 'survive' in the math courses at university and so she will continue her study. She felt a lack of positive stimuli for teacher students towards the intended later profession. So she demands fundamental changes for teacher education.

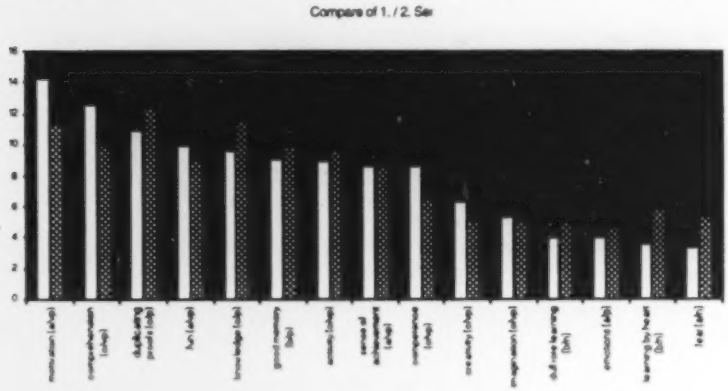
Mrs. E. Sec. II. At school she passed through advanced courses of mathematics. Her attitude towards mathematics was positive in general. She compare mathematics with an adventure playground. There is a slight, but significant change in her beliefs. She had failed the written examinations. Never before she had thought that mathematics is as hard as she experienced. Further, to her opinion, there are nearly no links to the intended profession. So, she claims for drastic changes.

Mr. L. Sec. II. He attended only basic courses at a school. His positive view towards mathematics was much influenced by teachers at school. To study mathematics was dominated by the fact to become a teacher as it is true for most of the teacher students. He successfully passed through the written examinations. Nevertheless there were moments when he asked himself whether he should quit his study of mathematics. He is complaining heavily on the circumstances of his math education at university. The university courses can be regarded as good counterexamples how students should never be treated. This observations lead to substantial increase in fear.

Some conclusions

The diagram below represents the means of the ranking. It is note suprising that variables with a very positive outcome score lower in the second series. This applies in particular to the variable 'motivation' studying mathematics. Further, affective components with a positive outcome score lower in the second interviews, however fear is in-

creasing. Also the high estimation of cognitive variables suffer losses in general; only the ranking of knowledge, duplicating of proofs as well as own activity and sense of achievement receive better scores. On the other hand, behavioral (conative) terms are ranked higher without exceptions.



All teacher students, the most successful as well as those we have immediately acquired "mathematics anxiety", ask for an integration of a practical training as early as possible. It would help them to prepare themselves for their later role as a teacher on one side and to improve and activate the actual learning processes at university on the other side. We got the impression that nearly each student fears that his undamaged view on mathematics which he has acquired at school is threatened by his experience during the first semester. So, they express their wish that mathematics might remain unhurt as they believed before: characterized by creativity, beauty and utility.

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